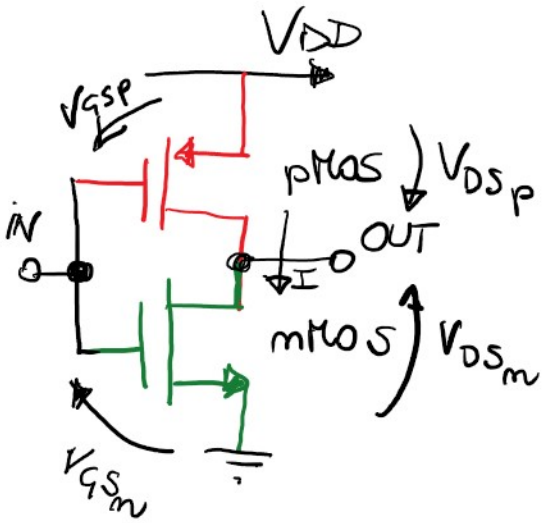


# Inverter CMOS

mercoledì 25 marzo 2020 10:34



HP  $V_{DD} > V_{Tn}$

$$V_{GS_m} = IN - 0 = IN.$$

$$V_{DS_m} = OUT - 0 = OUT$$

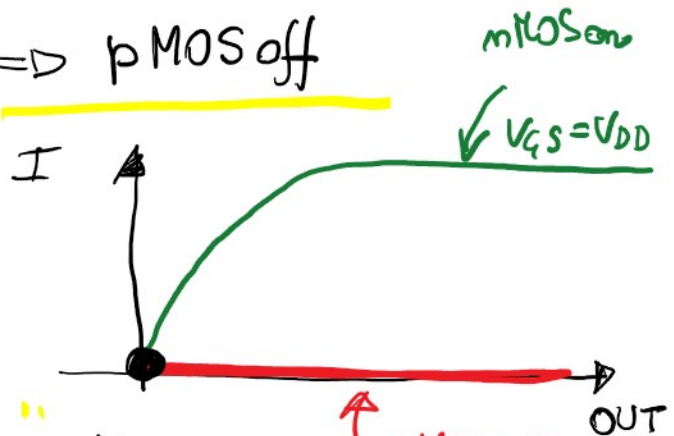
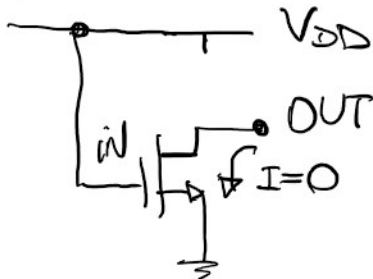
$$V_{GS_p} = IN - V_{DD}.$$

$$V_{DS_p} = OUT - V_{DD}$$

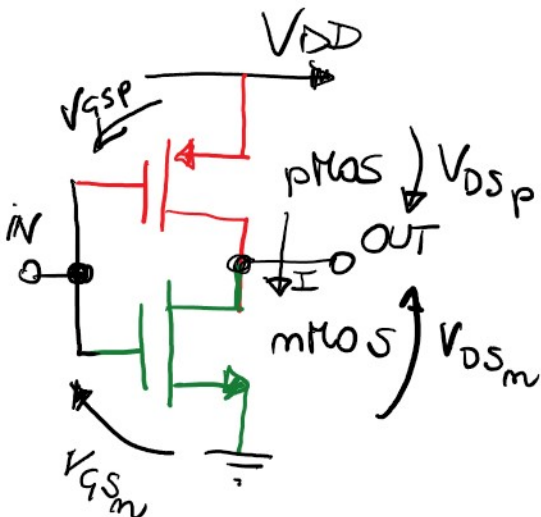
$$V_{Tn} = |V_{Tp}| = V_T.$$

•  $IN = '1'$   $\Rightarrow IN = V_{DD}$   
 $V_{GS_m} = V_{DD} > V_{Tn} \Rightarrow$  mTOS on

$$V_{GS_p} = V_{DD} - V_{DD} = 0 \Rightarrow$$
 pMOS off



$\hookrightarrow$  mTOS ohmico con  $V_{DS_m} = 0$   
 $\hookrightarrow$   $OUT = 0V \Rightarrow$   $OUT = '0'$  OK



$$V_{GS_p} = IN - V_{DD}$$

$$V_{DS_p} = OUT - V_{DD}$$

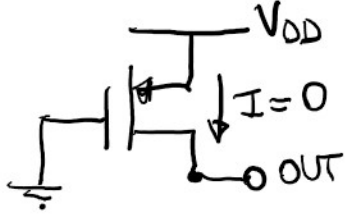
$$I = -I_{Dp}$$

$$OUT = V_{DS_p} + V_{DD}$$

$OUT = V_{Dsp} + V_{DD}$

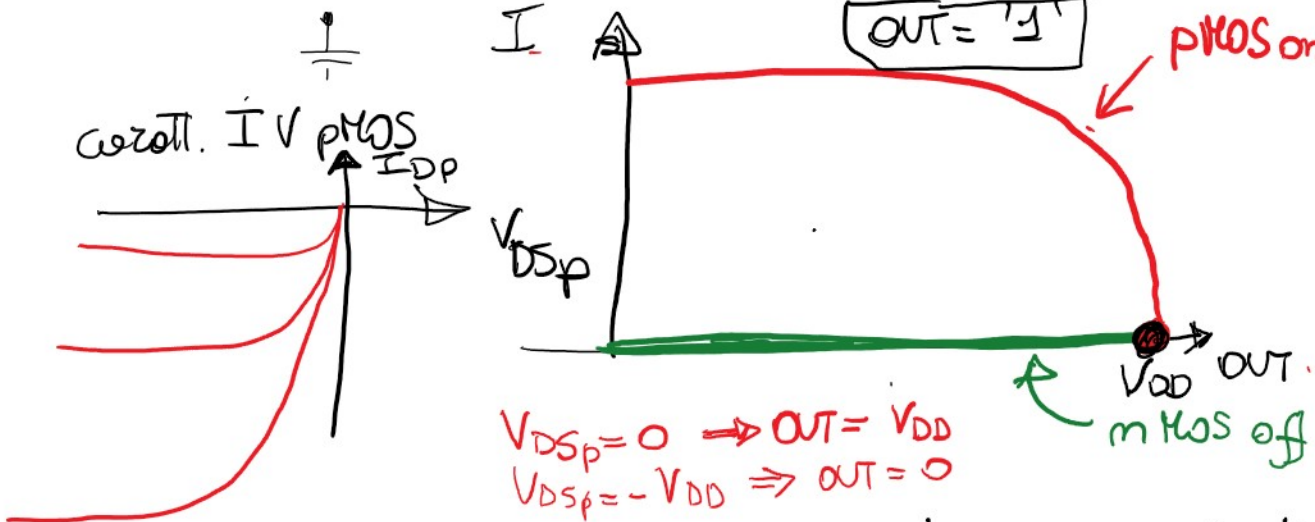
$V_{GS_m} = 0 \Rightarrow V_{GS_m} = 0 < V_{Tn} \Rightarrow$  mKOS off

$V_{GS_p} = 0 - V_{DD} = -V_{DD} < V_{Tp}$



$L \Rightarrow$  pKOS on  
 $I = 0$   
 $\rightarrow V_{Dsp} = 0$   
 $\rightarrow OUT = V_{DD}$

$OUT = '1'$  pKOS on

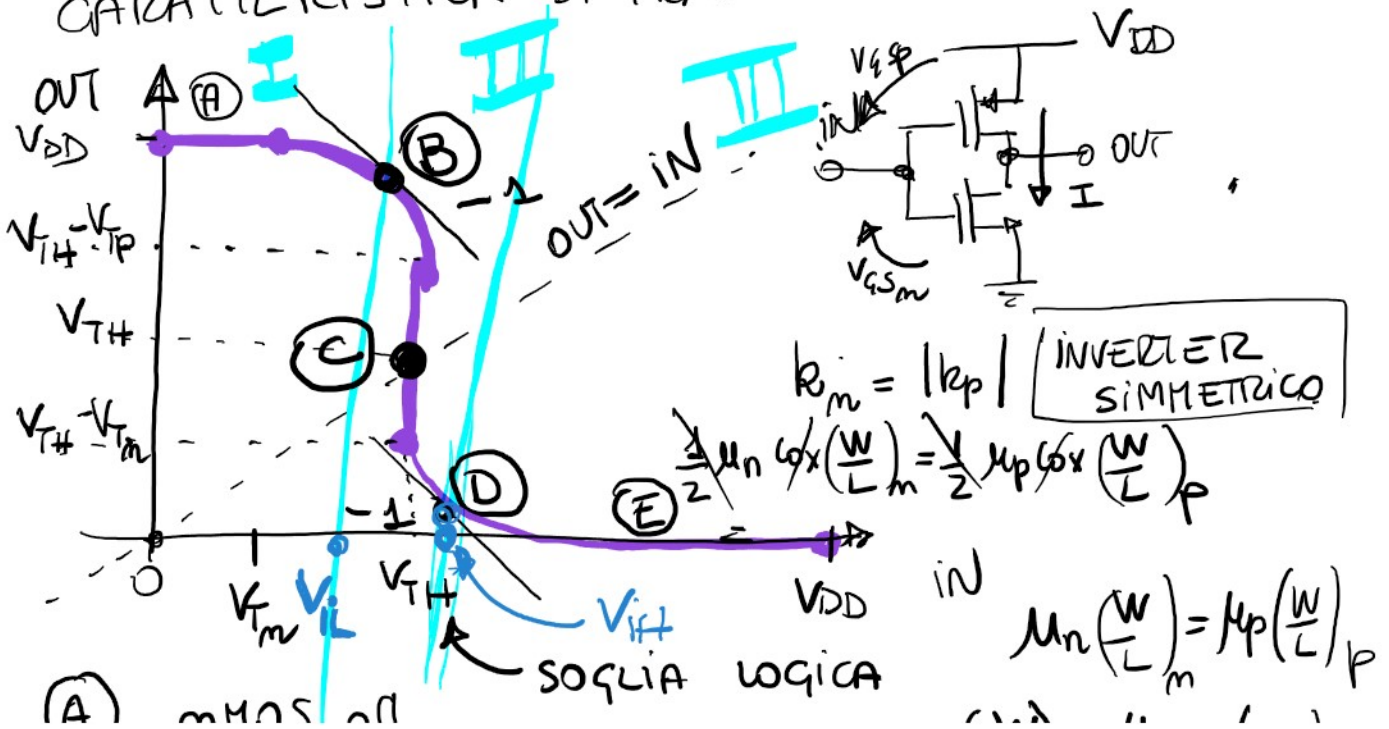


$I_{statica} = 0$  sia per  $IN = '0'$  che per  $IN = '1'$

$P_{STAT} = 0$

DISS. POTENZA  
 STATICA NULLA

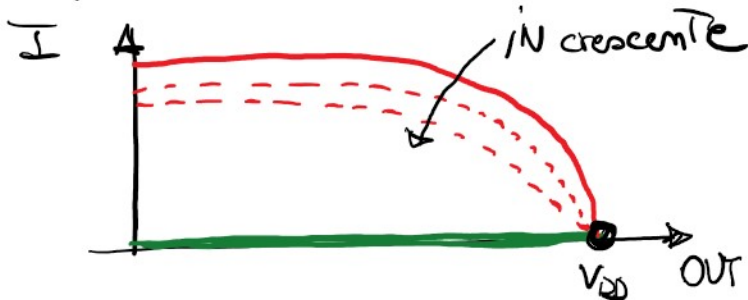
CARATTERISTICA DI TRASFERIMENTO STATICA





(A) mmos off  $\leftarrow$  SOGLIA LOGICA

pmos on in zona ohmica



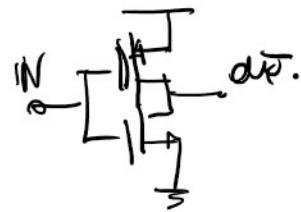
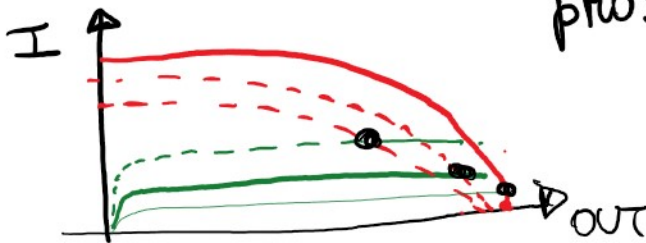
$$\left(\frac{W}{L}\right)_p = \frac{\mu_n}{\mu_p} \left(\frac{W}{L}\right)_m = 2.5 \left(\frac{W}{L}\right)_m$$

$$\mu_n \approx 800 \text{ cm}^2/\text{Vs}$$

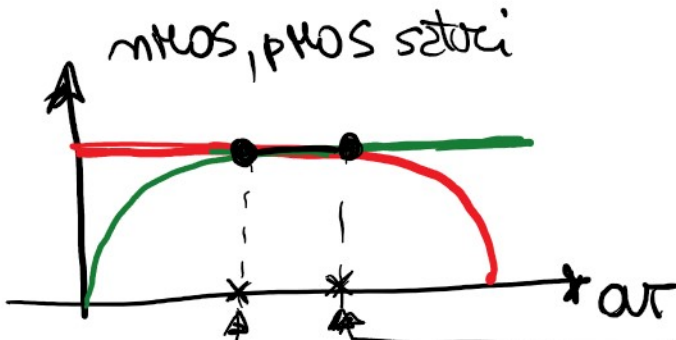
$$\mu_p \approx 300 \text{ cm}^2/\text{Vs}$$

(B) mmos on in zona di saturazione

pmos on in zona ohmica



(C)



$$V_{DS_m} = V_{GS_m} - V_T = V_{TH} - V_{Tm}$$

pmos

$$V_{DS_p} = V_{GS_p} - V_{Tp}$$

$$OUT - V_{DD} = IN - V_{DD} - V_{Tp}$$

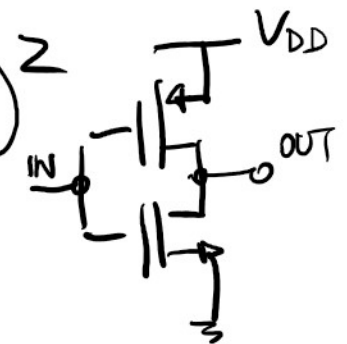
$$OUT = V_{TH} - V_{Tp}$$

CALCOLO DELLA SOGLIA LOGICA

$$I_{Dm_{SAT}} = |I_{Dp_{SAT}}|$$

$$k_m (V_{GS_m} - V_{Tm})^2 = |k_p| (V_{GS_p} - V_{Tp})^2$$

inverter simmetrico  $k_m = |k_p|$



$$(IN - V_{Tm})^2 = [(IN - V_{DD}) - V_{Tp}]^2$$

$$(I_N - V_{Tn}) = \left[ (I_N - V_{DD}) - V_{Tp} \right]$$

$$V_{Tn} = |V_{Tp}| = V_T > 0$$

$$(I_N - V_T)^2 = \left[ (I_N - V_{DD}) + V_T \right]^2$$

estraggo le radici quadrate

$$I_N - V_T = - \left[ (I_N - V_{DD}) + V_T \right]$$

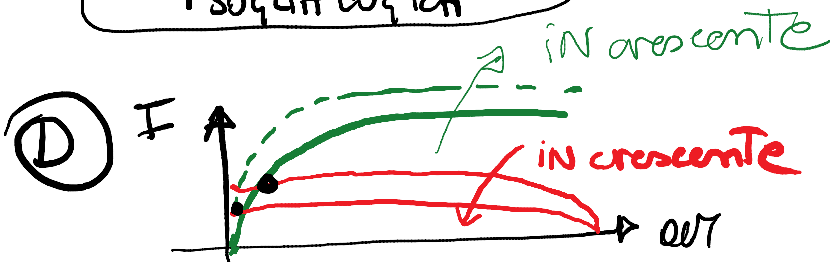
$$I_N - \cancel{V_T} = -I_N + V_{DD} - \cancel{V_T}$$

$$2I_N = V_{DD}$$

$$\boxed{I_N \triangleq V_{IH} = \frac{V_{DD}}{2}}$$

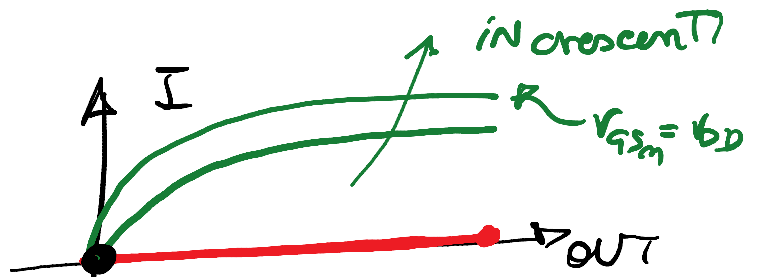
SOGLIA LOGICA

• SOGLIA LOGICA  
DI COMMUTAZIONE



mMOS ohmico  
pMOS saturo

② pMOS off  
mMOS ohmico

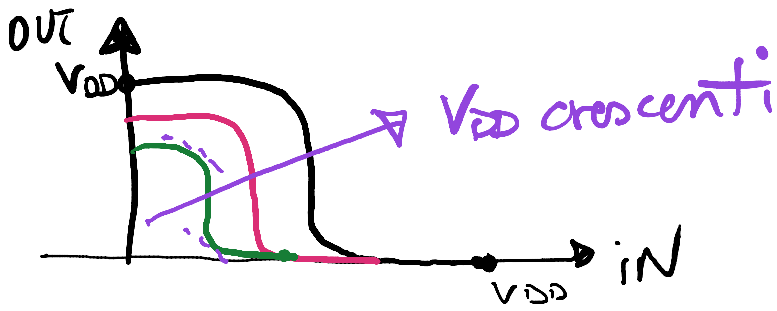


$V_{IL}$  = max IN interpretata come livello logico basso

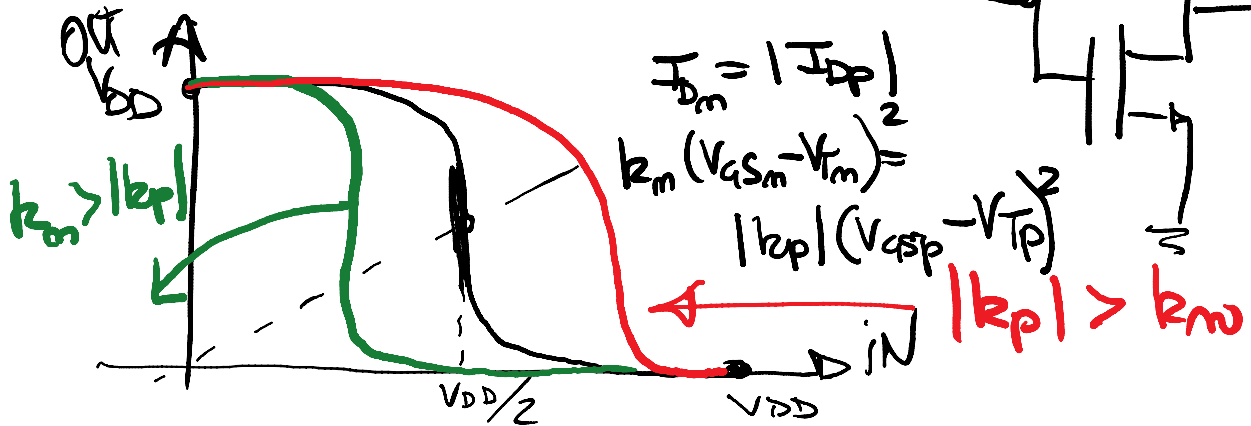
$V_{IH}$  = min IN interpretata come livello logico alto.

EFFETTO DELLA TENS. DI ALIMENTAZ.

# EFFETTO DELLA TENS. DI ALIMENTAZ.



## INVERTER NON SIMMETRICO

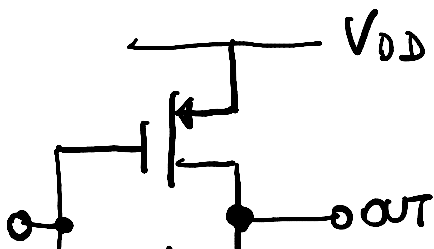


## INVERTER AREA MINIMA

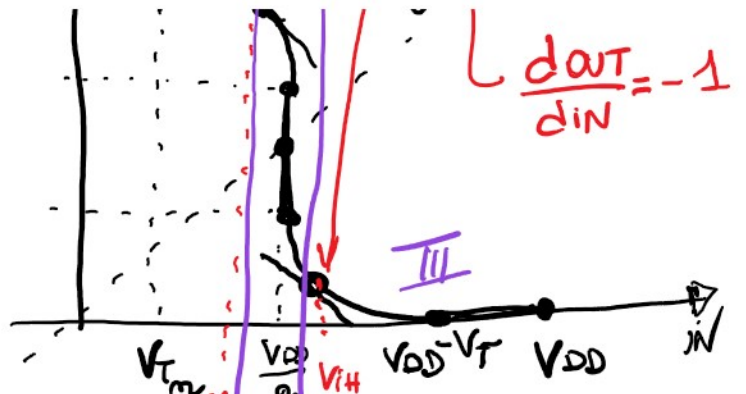
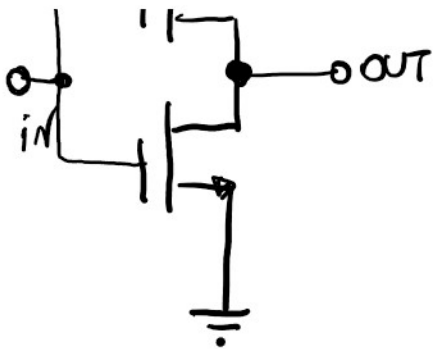
$$\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_n$$

$|k_p| \neq k_n \quad |k_p| = \frac{k_n}{2.5}$  ← rapporto di mobilità

## INVERTER CMOS - MARGINI DI RUMORE (NOISE MARGINS)







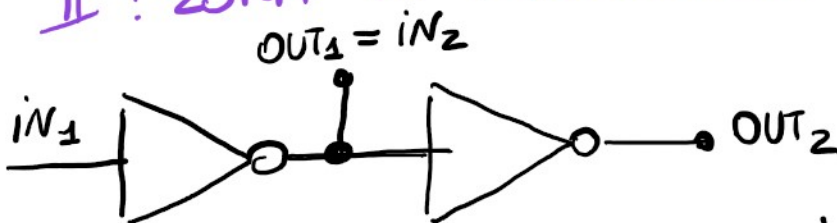
$$V_{T_m} = |K_{Tp}| = V_T$$

$$k_m = |k_p| \quad \text{INVERTER SIMMETRICO} \quad \left(\frac{W}{L}\right)_p = 2.5 \left(\frac{W}{L}\right)_m$$

$V_{iL}$  tale che  $\forall IN < V_{iL} \Rightarrow OUT = '1'$

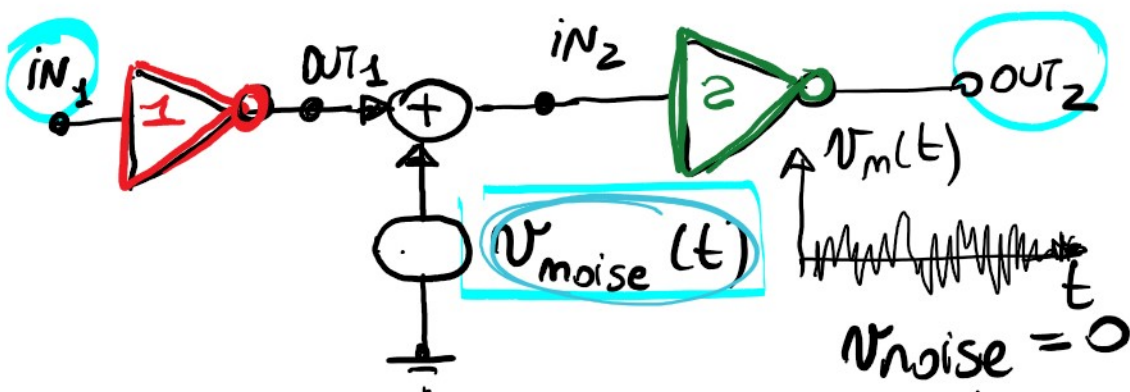
$V_{iH}$  tale che  $\forall IN > V_{iH} \Rightarrow OUT = '0'$

## II : ZONA DI TRANSIZIONE



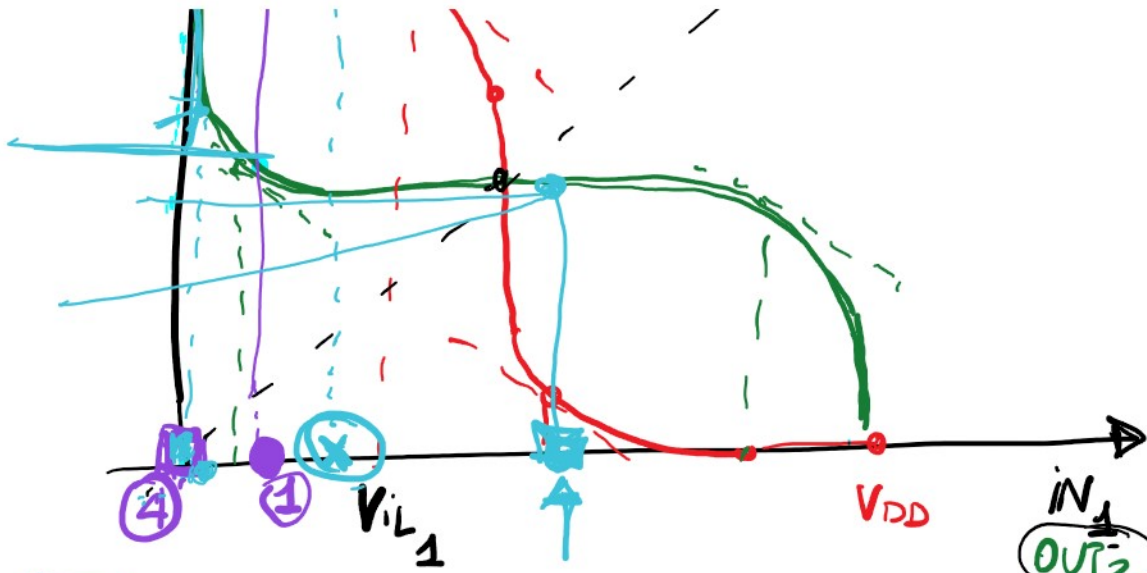
$$IN_1 = '1' \Rightarrow OUT_1 = '0' \Rightarrow IN_2 = '0' \Rightarrow OUT_2 = '1'$$

$$\overline{OUT_2} = \overline{IN_2} = \overline{OUT_1} = \overline{IN_1} = IN_1$$



$$IN_2 = OUT_1 + V_{noise}(t) \Rightarrow OUT_2 = IN_1$$





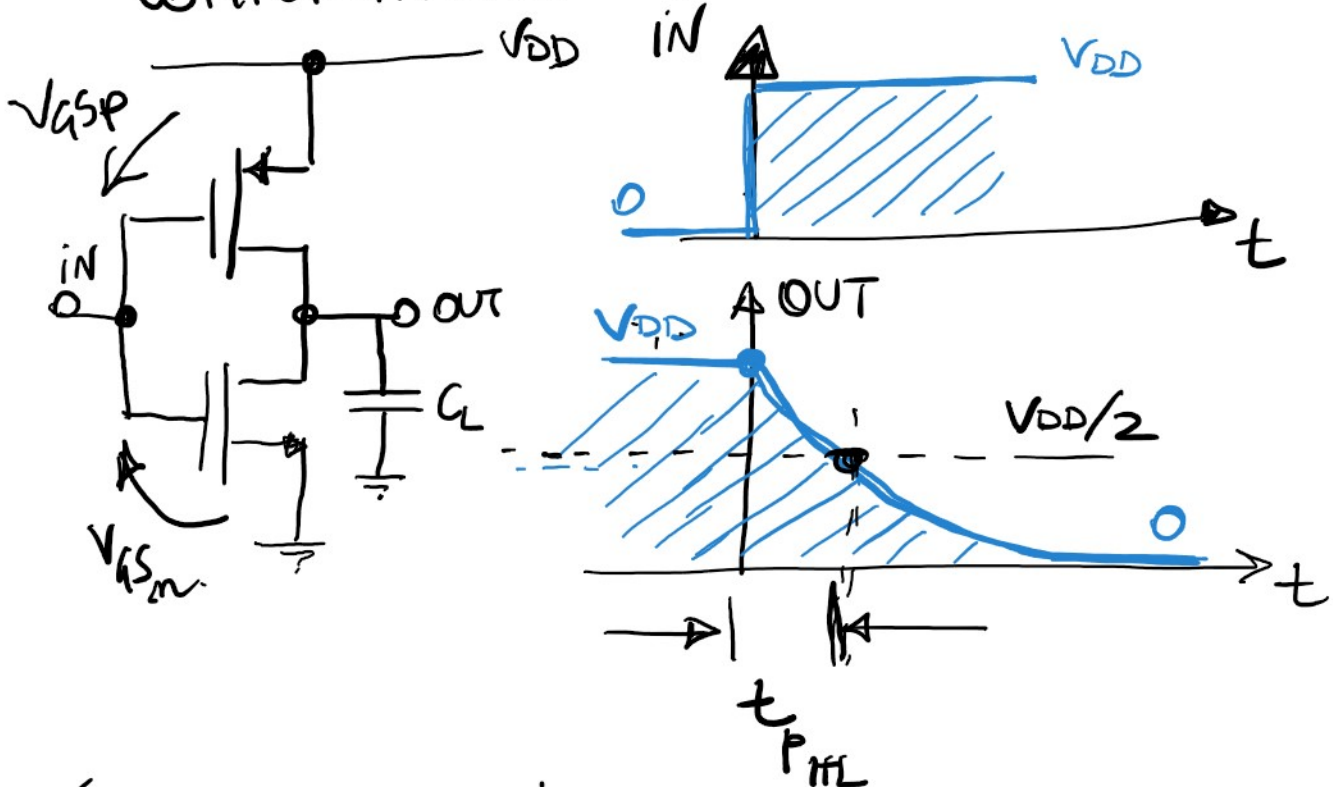
$$NM_L \triangleq V_{IL} - 0 = V_{IL} \quad \text{noise margin low}$$

$$NM_H \triangleq V_{DD} - V_{IH} \quad \text{noise margin high}$$

inverter simmetrico

$$NM_L = NM_H$$

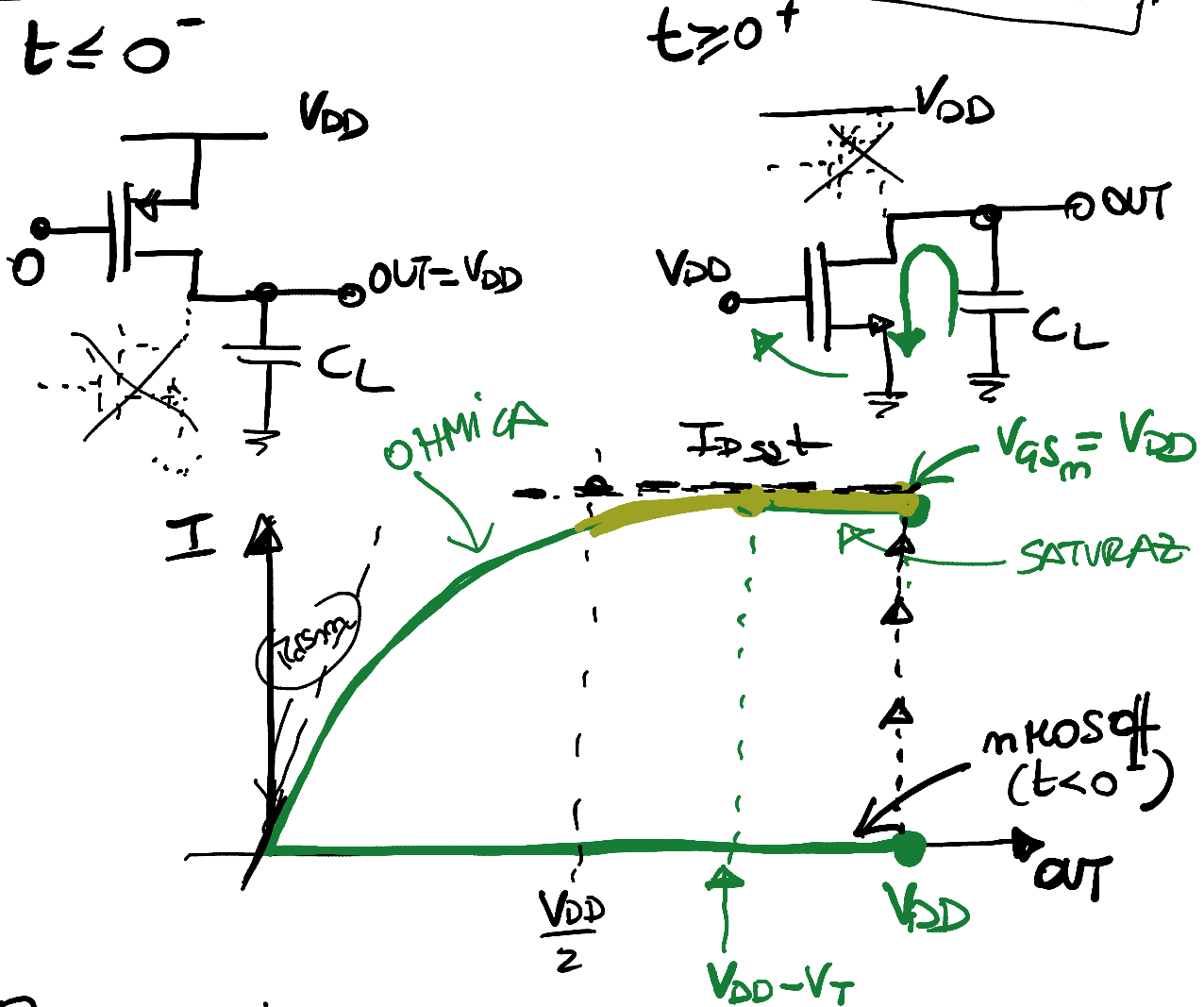
### COMPORIAMENTO DINAMICO DELL' INVERTER CMOS



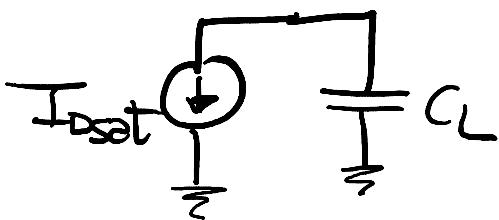
$t_{PHL}$  : TEMPO DI PROPAGAZIONE ALTO-BASSO:  
 tempo necessario perché l'uscita raggiunga  
 il 50% del suo valore ( $V_{DD}/2$ ) .

ga il 50% del suo valore ( $V_{DD}/2$ ) :

$t_{pLH}$  : TEMPO DI PROPAGAZIONE BASSO-ALTO  
 tempo necessario perché  $V_{out} = \frac{V_{DD}}{2}$



APPROSSIMAZIONE SATURA



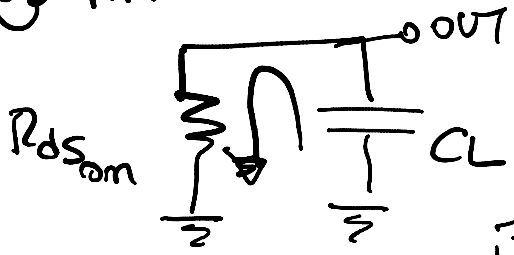
$$I_{Dsat} = k_m (V_{gs} - V_{Tm})^2 = k_m (V_{DD} - V_{Tm})^2$$

$$t_{pHL} = \frac{Q_{50\%}}{I_{Dsat}} = \frac{C_L (V_{DD} - V_{DD}/2)}{k_m (V_{DD} - V_{Tm})^2} = \frac{C_L V_{DD}/2}{k_m (V_{DD} - V_{Tm})^2}$$



$$= \frac{C_L V_{DD}/2}{k_m (V_{DD} - V_{Tn})^2}$$

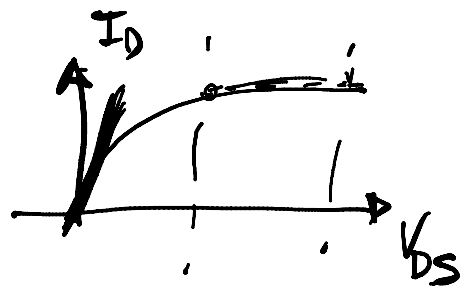
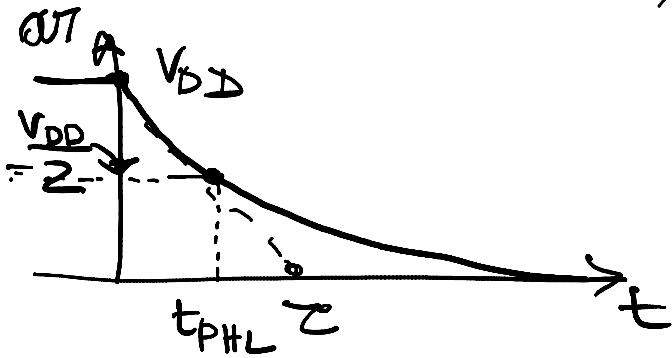
### APPROSSIMAZIONE OHMICA



$$\tau = R_{DS_{om}} C_L$$

$$R_{DS_{om}} \triangleq \left. \frac{\partial V_{DS}}{\partial I_D} \right|_{V_{DS}=0} = \frac{1}{2k_m (V_{DD} - V_{Tn})}$$

$$OUT = V_{DD} \exp(-t/\tau)$$



$$\frac{V_{DD}}{2} = V_{DD} \exp\left(-\frac{t_{pHL}}{\tau}\right)$$

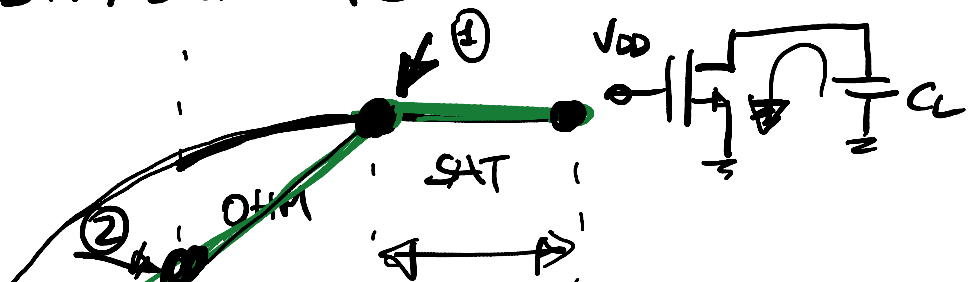
$$\frac{1}{2} = \exp\left(-\frac{t_{pHL}}{\tau}\right)$$

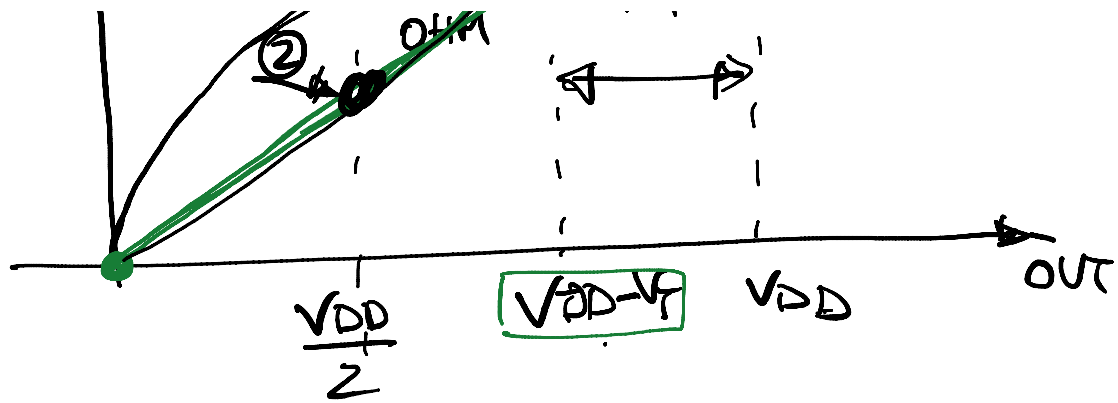
$$\ln \frac{1}{2} = \ln \left[ \exp\left(-\frac{t_{pHL}}{\tau}\right) \right]$$

$$\ln \frac{1}{2} = -\frac{t_{pHL}}{\tau}$$

$$t_{pHL} = \tau \ln 2 = 0.69 \tau \quad (3)$$

### APPROSSIMAZIONE PER ECCESSO





$$t_{pHL} = t_{pHL_{sat}} + t_{pHL_{OHM}}$$

$$t_{pHL_{sat}} = \frac{Q(V_{DD} - (V_{DD} - V_T))}{I_{Dsat}} = \frac{C_L (V_{DD} - V_{DD} + V_T)}{I_{Dsat}}$$

$$I_{Dsat} = k_m (V_{GS} - V_{Tm})^2 = k_m (V_{DD} - V_{Tm})^2$$

$$= \frac{C_L V_T}{k_m (V_{DD} - V_{Tm})^2}$$

A  $t_{pHL_{OHM}}$

$$R_{eq} = \frac{[V_{DD} - V_T - 0]}{I_{Dsat}} = \frac{V_{DD} - V_T}{k_m (V_{DD} - V_{Tm})^2}$$

$$\tau = C_L R_{eq}$$

$$OUT = (V_{DD} - V_T) \exp(-t/\tau)$$

$$\frac{V_{DD}}{2} = (V_{DD} - V_T) \exp(-t/\tau)$$

$$\frac{\frac{V_{DD}}{2}}{V_{DD} - V_T} = \exp(-t/\tau)$$

$$\ln \frac{V_{DD}/2}{V_{DD}-V_T} = - \frac{t_{pHL_{ohm}}}{\tau}$$

$$t_{pHL_{ohm}} = \tau \ln \frac{V_{DD}-V_T}{V_{DD}/2}$$

$$\Downarrow t_{pHL} = t_{pHL_{ohm}} + t_{pHL_{saturo.}}$$

$$\tau_p \approx \frac{t_{pHL} + t_{pLH}}{2}$$

inverter  
simmetrica

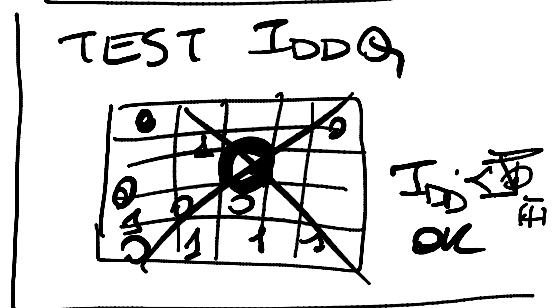
$$t_{pHL} = t_{pLH}$$

RITARDO DI PROPAGAZIONE

## DISSIPAZIONE DI POTENZA INVERTER CMOS

★ POTENZA STATICA

$$P_{STATICA} = 0$$

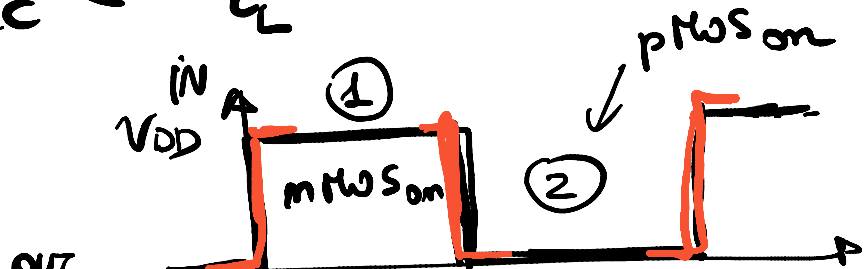
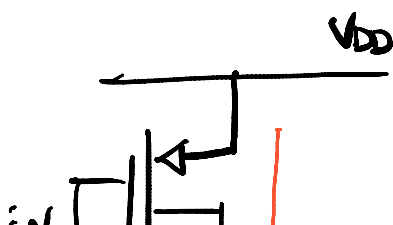


★ POTENZA DINAMICA

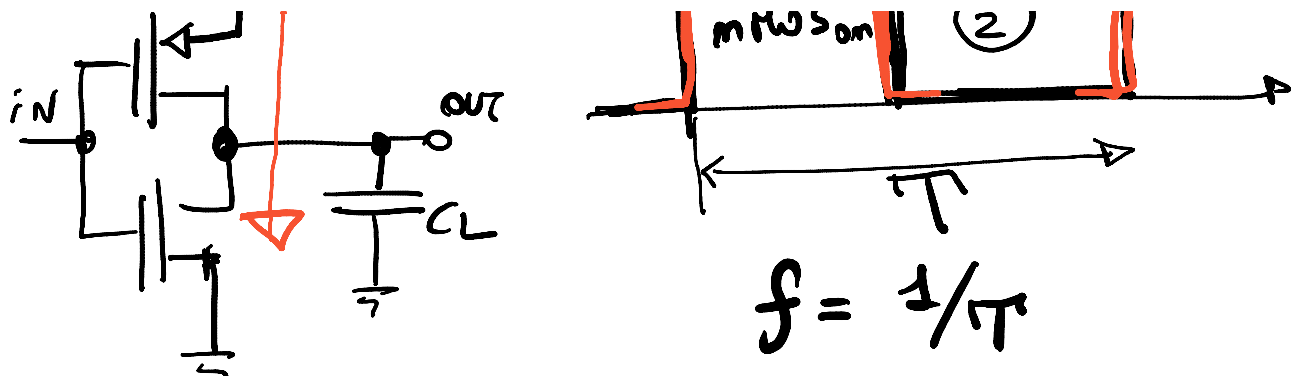
○ Potenza spesa per lo carica e scarica della capacità ( $P_C$ )

○ Potenza di cross-conduzione ( $P_{CC}$ )

$$P_{CC} \ll P_C$$







①  $C_L$  è carico di  $V_{DD}$   
 $E = \frac{1}{2} C_L V_{DD}^2$   
 Durante lo scarica e è dissipata dall' mTOS

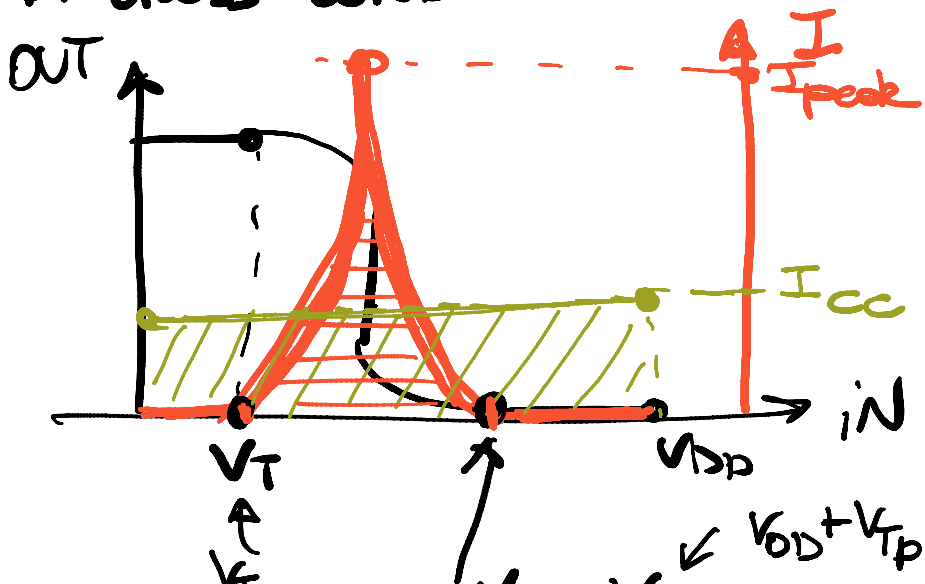
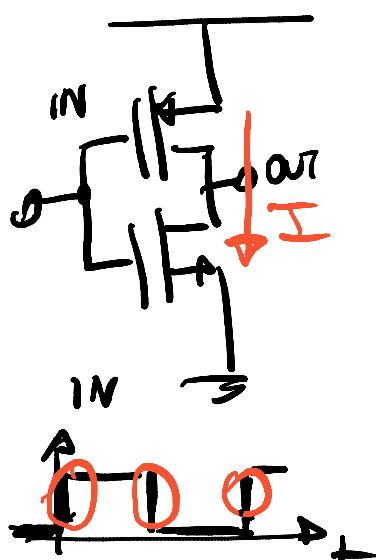
②  $E = \int i(t) v(t) dt =$   
 $= V_{DD} \int i(t) dt = V_{DD} C_L V_{DD} =$   
 $= C_L V_{DD}^2$

Annotations:  
 -  $\frac{1}{2} C_L V_{DD}^2$  in  $C_L$   
 -  $\frac{1}{2} C_L V_{DD}^2$  dissipata dal pTOS.

$$P_{C_L} = C_L V_{DD}^2 f$$

ridurre le tensioni di element.

ridurre tutti i percentismi  
**POTENZA DI CROSS-CONDUZIONE**





$V_{Tn}$

$V_{DD} - V_T$   $V_{DD} + V_{Tp}$

$$I_{peak} = k_m (V_{GS} - V_{Tn})^2 = k_m \left( \frac{V_{DD}}{2} - V_{Tn} \right)^2 = |k_p| \left( -\frac{V_{DD}}{2} - V_{Tp} \right)^2$$

$$P_{CC} = I_{CC} V = (*)$$

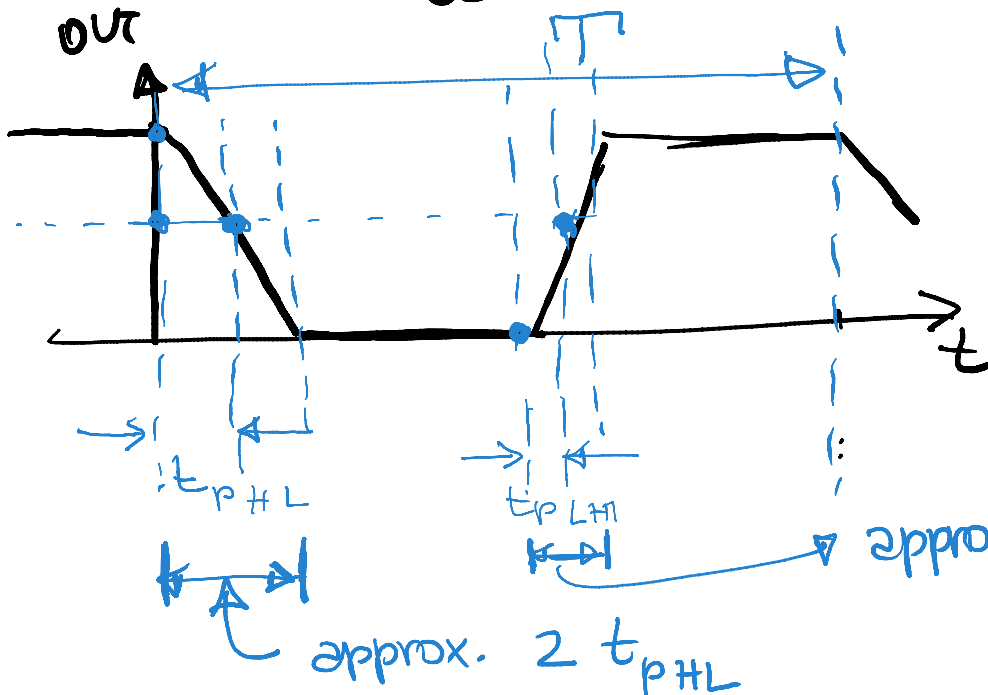
$$\frac{1}{2} (V_{DD} - V_T - V_T) * I_{peak} = I_{CC} V_{DD}$$

$$I_{CC} = \frac{1}{2} I_{peak} \frac{V_{DD} - V_{Tn} - V_{Tp}}{V_{DD}}$$

corrente di cross-conduzione media

$$(*) = \left( \frac{1}{2} I_{peak} \frac{V_{DD} - 2V_T}{V_{DD}} \right) V_{DD} \frac{2t_{pHL} + 2t_{pLH}}{T} =$$

$I_{CC}$



$$t_{pHL} + t_{pLH} = 2\tau_p$$

ritardo di propag.

$$P_{CC} = \frac{1}{9} I_{peak} (V_{DD} - 2V_T) 2(2\tau_p) f$$

$$I_{CC} = \frac{1}{2} I_{peak} \quad (1000 - \dots) \quad \approx (4-5) J$$

$$P_{CC} \ll P_{CL}$$

$$P_{TOT} = \cancel{P_{STATICA}} + P_{CC} + P_{CL}$$