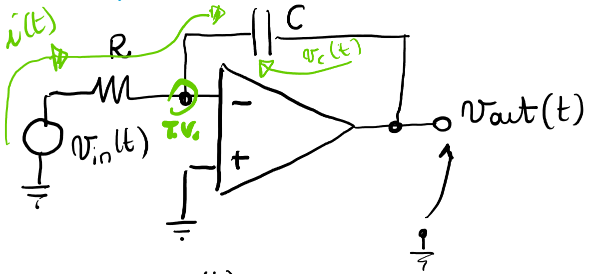
Lezione 16b: Integratore di Miller e approssimato, derivatore ideale e approssimato. Larghezza di banda finita dell'opamp.

martedì 12 maggio 2020 09:31

INTEGRATORE DI MILLER



$$i(t) = \frac{v_{in}(t)}{R}$$

$$Q(t) = \int_0^t i(\tau)\,d\tau$$ carica depositata sulle armature di C

$$v_c(t) = V_c + \frac{Q(t)}{C} = V_c + \frac{\int_0^t i(\tau)\,d\tau}{C} =$$

$$= V_c + \frac{1}{RC}\int_0^t v_{in}(\tau)\,d\tau$$

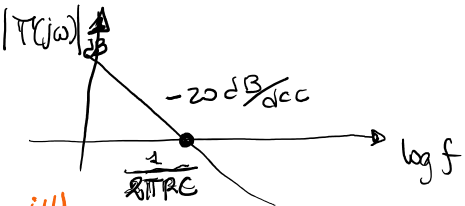
$$\boxed{v_{out}(t) = -v_c(t) = -V_c - \frac{1}{RC}\int_0^t v_{in}(\tau)\,d\tau}$$

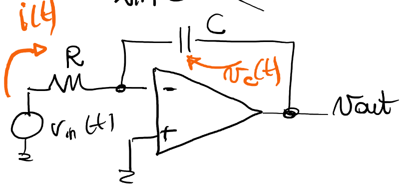
COSTANTE DI TEMPO DI INTEGRAZIONE

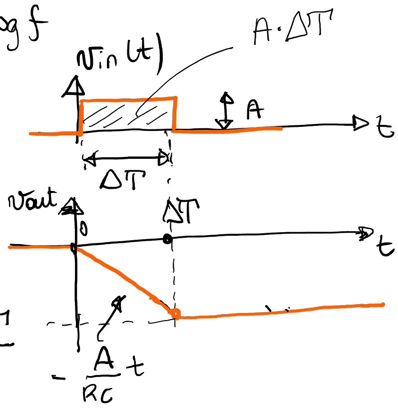
$$Z_1(s) = R\ ;\quad Z_2(s) = \frac{1}{sC}$$

$$T(s) \triangleq \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{1}{sRC}$$

$$|T(j\omega)| = \frac{1}{\omega RC}$$





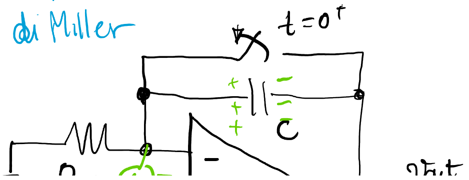


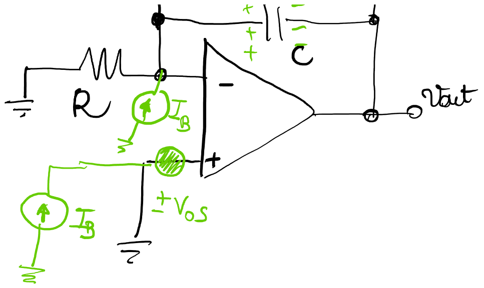
$$i_c = C\frac{dv_c(t)}{dt}$$

$$\frac{dv_c(t)}{dt} = \frac{i_c}{C} =$$

$$= \frac{A}{R}\frac{1}{C}$$

Effetto delle non idealità dell'opamp sull'integratore di Miller





$t \leq 0^-$ $\qquad v_{out}(t) = \pm V_{OS}$

$t = 0^+$ l'interruttore si apre

applichiamo il princ. di sovrapposizione degli effetti

$\underline{V_{OS}|}$

$$v_{out}\Big|_{V_{OS}} = \pm V_{OS} + \frac{\int_0^t \overbrace{\frac{\pm V_{OS}}{R}}^{i(\tau)} d\tau}{C} = \pm V_{OS} \pm \frac{V_{OS}}{RC} t$$

$\underline{I_B|}$ $\quad I_B$ al $\oplus$ non dà contributo

I_B al $\ominus$; $v^+ = 0 \Rightarrow v^- = 0$ (terra virtuale) in R non scorre corrente

$$v_{out}(t)\Big|_{I_B} = -\frac{Q(t)}{C} = -\frac{\int_0^t I_B d\tau}{C} = -\frac{I_B t}{C}$$

$\Downarrow$

$$v_{out}(t) = \pm V_{OS} \pm \frac{V_{OS}}{RC} t - \frac{I_B}{C} t$$

dopo quanto tempo l'uscita satura ($V_{out} = V_{out}|_{SAT}$)

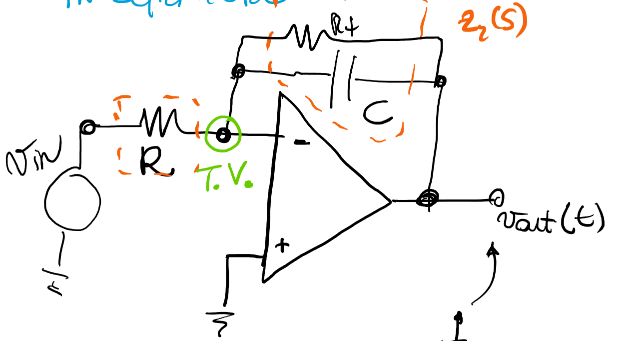
$|V_{out}|_{SAT}| = 10V$

$R = 10k\Omega$
$C = 100pF$
$V_{OS} = \pm 1.5mV$
$I_B = 100nA$

$$\Delta t = \frac{V_{OUT}|_{SAT} \mp V_{OS}}{\pm\frac{V_{OS}}{RC} - \frac{I_B}{C}} \cong \frac{-10V}{-\frac{1.5mV}{\underset{10^4}{10k} * \underset{10^{-10}}{100p}} - \frac{100nA}{100p}} =$$

$$= \frac{10}{1.5\,10^3 + 10^3} = \frac{10}{2.5\ 10^3} \simeq 4\,ms\ !!$$

INTEGRATORE APPROSSIMATO

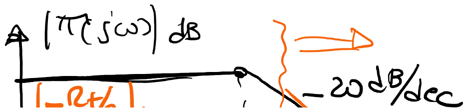


$$Z_2 = \frac{R_f}{1+sCR_f}$$

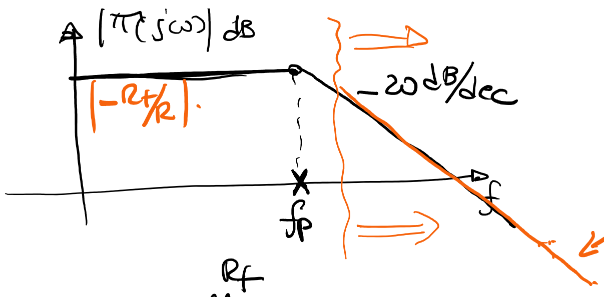
$$Z_1 = R$$

$$T(s) \triangleq \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)} = \underbrace{-\frac{R_f}{R}}_{T(0)} \cdot \frac{1}{1+sCR_f}$$

f_p



$f_p = \frac{1}{\ }$

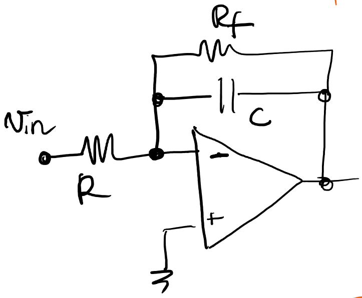


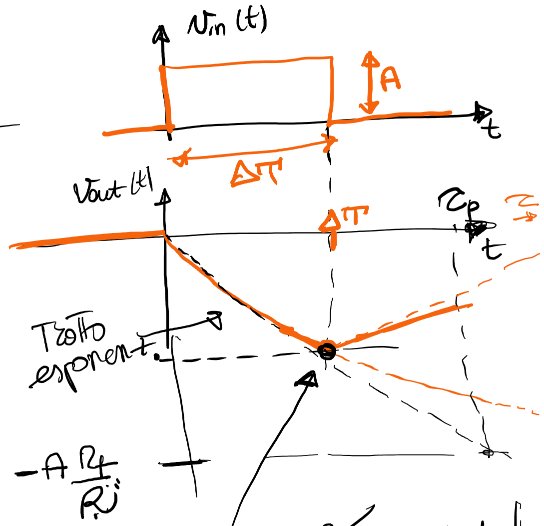
$$f_p = \frac{1}{2\pi\tau_p}$$

$$\tau_p = C R_f$$

INTEGRATORE

$f \gg f_p$

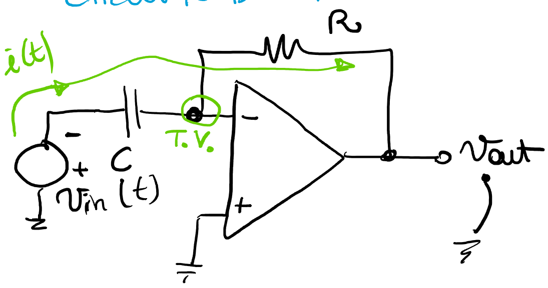




$$-A\frac{R_f}{R}\cdot\frac{1}{\tau}t = -A\frac{\cancel{R_f}}{R}\cdot\frac{1}{C\cancel{R_f}}t\Big|_{t=\Delta T}$$

$$= -\frac{A}{RC}\Delta T$$

CIRCUITO DERIVATORE IDEALE



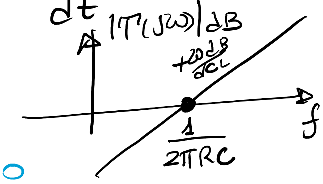
Analisi nel dominio del tempo

$$i(t) = C\frac{dv_C}{dt} = C\frac{dv_{in}(t)}{dt}$$

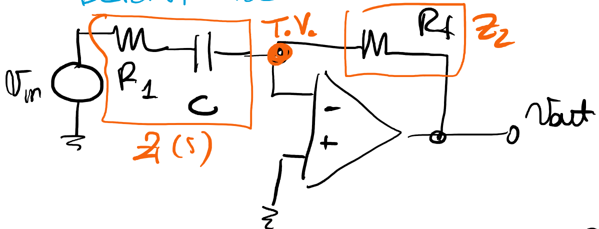
COSTANTE DI TEMPO DI DERIVAZIONE

$$v_{out}(t) = -i(t)R = -RC\frac{dv_{in}}{dt}(t)$$

$$T(s) = -\frac{R}{1/sC} = -sRC$$



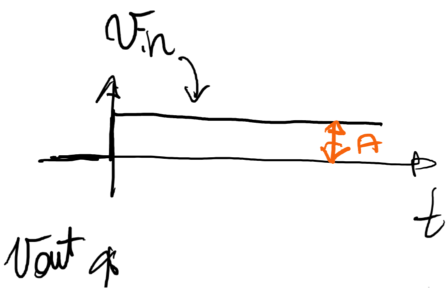
DERIVATORE APPROSSIMATO



$$Z_2(s) = R_f$$

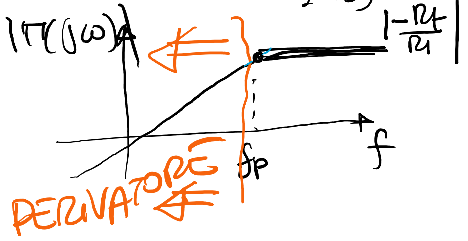
$$Z_1(s) = R_1 + \frac{1}{sC} = \frac{R_1 sC + 1}{sC}$$

$$T(s) = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R_f sC}{1 + sCR_1}$$

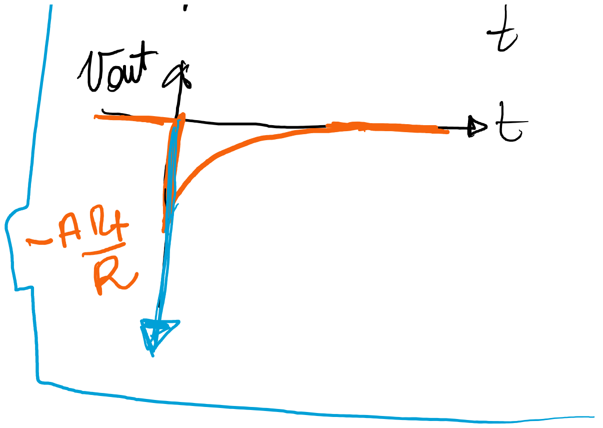


$$T(s) = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R_4\, sC}{1+sCR_1}$$

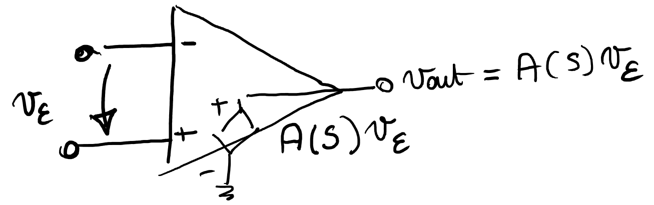
$\tau_p = CR_1$

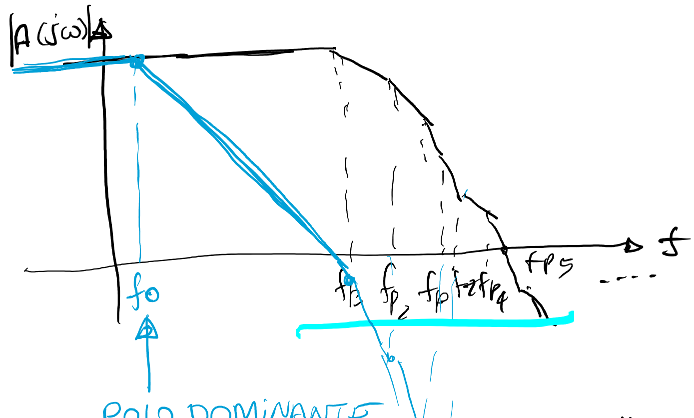


$$-\frac{\cancel{s}\cancel{C}R_4}{\cancel{s}\cancel{C}R_1} = -\frac{R_4}{R_1}$$



RISPOSTA IN FREQUENZA E LARGHEZZA DI BANDA DELL'AMPLIFICATORE OPERAZIONALE





$$A(s) = \frac{A_0}{1+s\tau_0}$$

A_0: guadagno ad anello aperto in continua dell'opamp

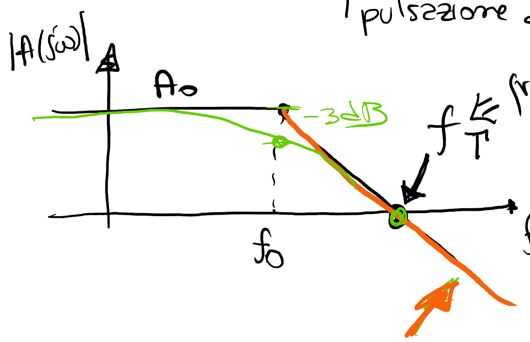
τ_0: COSTANTE DI TEMPO AD ANELLO APERTO DELL'OPAMP

$$A(j\omega) = \frac{A_0}{1+j\omega\tau_0} = \frac{A_0}{1+j\frac{\omega}{\omega_0}}$$

$$|A(j\omega)| = \frac{A_0}{\sqrt{1+\frac{\omega^2}{\omega_0^2}}} \qquad \omega_0 = \frac{1}{\tau_0}$$

A_0: guadagno in continua ad anello aperto

ω_0: pulsazione a $-3dB$ (ad anello aperto)



f_T ← frequenza di taglio

$$f_0 = \frac{1}{2\pi\tau_0}$$

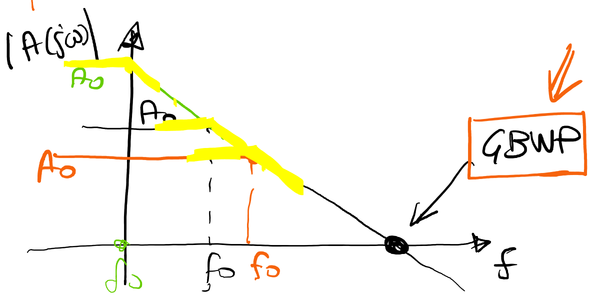
$$|A(j\omega)| \underset{\omega\gg\omega_0}{\approx} \frac{A_0}{\sqrt{\frac{\omega^2}{\omega_0^2}}} = \frac{A_0\,\omega_0}{\omega}$$

$@\ \omega_T = 2\pi f_T \qquad |A(j\omega_T)| = 1$

@ $\omega_T = 2\pi f_T$ $\qquad |A(j\omega_T)| = 1$

$$1 = \frac{A_0\,\omega_0}{\omega_T} \Rightarrow \omega_T = \underbrace{A_0\,\omega_0} = GBWP$$

GBWP
Gain
Bandwidth
Product



$A_0 \qquad 10^5 \div 10^7$ typ

$f_0 \qquad 10 - 100$ Hz

$A_0 f_0 = 10^6 \div 10^9 \qquad GBWP = 1\,\text{MHz} \div$

$= 1\,\text{GHz}$

$GBWP \simeq 10\,\text{GHz}$