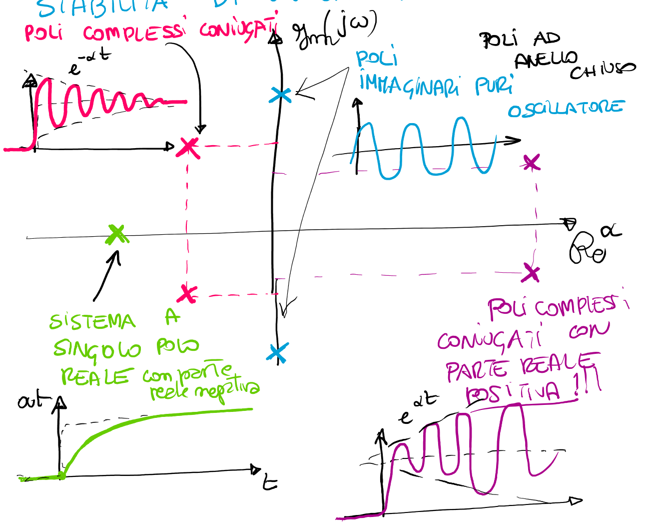
STABILITÀ DI UN CIRCUITO RETROAZIONATO



$$G_{loop}(s) = \frac{G_{loop}(0)}{(1+s\tau_1)(1+s\tau_2)}$$

$G_{loop}(0) < 0$ sistema retroazionato negativamente

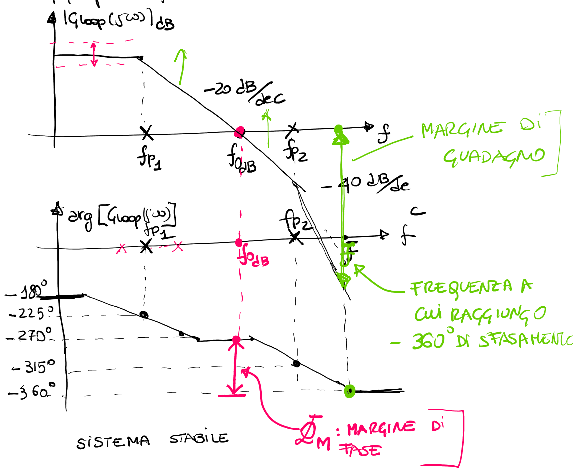
$$f_{p_1} = \frac{1}{2\pi\tau_1} \qquad f_{p_2} = \frac{1}{2\pi\tau_2}$$

$\bar{\omega}$ a cui lo sfasamento raggiunge $360°$

(A) • $|G_{loop}(j\bar{\omega})| < 1 \Rightarrow$ SISTEMA STABILE

• $|G_{loop}(j\bar{\omega})| = 1 \Rightarrow$ OSCILLATORE PURO

(B) • $|G_{loop}(j\bar{\omega})| > 1 \Rightarrow$ SISTEMA INSTABILE



$$\Phi_M = \arg\left[G_{loop}(j\omega_{0dB})\right] - (-360°)$$

$$\omega_{0dB} = 2\pi f_{0dB}$$

$\Phi_M > 0$ SISTEMA STABILE

$\Phi_M > 45°$ CONDIZIONE PER LA STABILITÀ

$|G_{loop}(j\omega)| = 1$ $\qquad \Phi_M = 45°$

$G_{loop}(j\omega) = 1 \cdot e^{-j\theta}$ $\qquad \Downarrow \ \theta = -315°$

$\Downarrow$

$$G_{reale} = \frac{G_{andata}}{1 - G_{loop}} = \frac{G_{andata}}{1 - 1\,e^{-j\theta}} =$$

$$G_{andata} = -G_{id} * G_{loop}$$

$$= \frac{-G_{ideale}\,G_{loop}}{1 - e^{-j\theta}} = \frac{-G_{ideale}\,1 \cdot e^{-j\theta}}{1 - 1e^{-j\theta}} =$$

Calcolo il modulo di G_{reale}:

$$|G_{reale}| = |G_{ideale}| \left| \frac{e^{-j\theta}}{1 - e^{-j\theta}} \right|$$

$$e^{-j\theta} = \cos\theta + j\sin\theta$$

$$\left| \frac{e^{-j\theta}}{1 - e^{-j\theta}} \right| = \left| \frac{\cos\theta + j\sin\theta}{1 - \cos\theta - j\sin\theta} \right| =$$

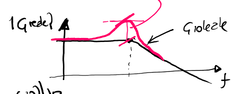
$e^{-j\theta} = \cos\theta + j\sin\theta'$

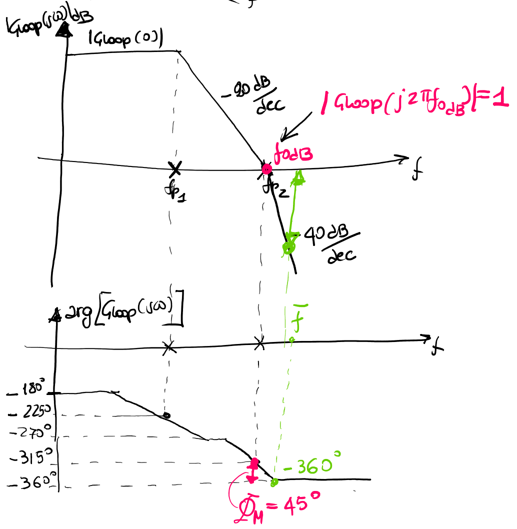
$$\left|\frac{e^{-j\theta}}{1-e^{-j\theta}}\right| = \left|\frac{\cos\theta + j\sin\theta}{1-\cos\theta - j\sin\theta}\right| =$$

$$= \frac{\sqrt{\cos^2\theta+\sin^2\theta}}{\sqrt{(1-\cos\theta)^2+\sin^2\theta}} = \frac{1}{\sqrt{1-2\cos\theta+\underbrace{\cos^2\theta+\sin^2\theta}}} =$$

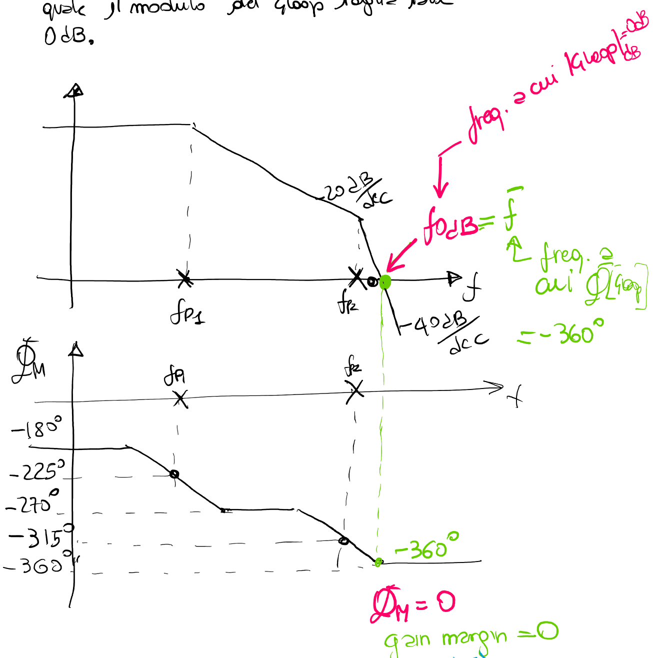
$$= \frac{1}{\sqrt{2-2\cos\theta}} = \frac{1}{\sqrt{2}\,\sqrt{\underbrace{1-\cos(-315^\circ)}_{0.29}}} =$$

$= 1.306$ 30.6% !!





MARGINE DI FASE: complemento a (-360°) della fase di Gloop alla frequenza alla quale il modulo del Gloop Taglia l'asse 0dB.



CRITERI DI BODE PER LA STABILITÀ
(CONDIZIONE SUFFICIENTE)

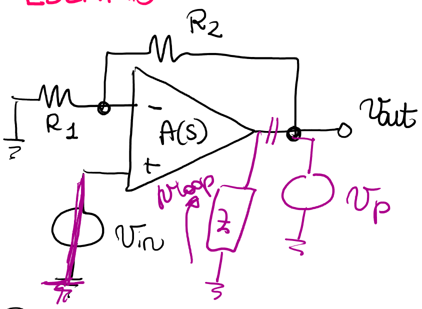
- |Gloop(jω)| Taglia l'asse 0dB con pendenza -20 dB/dec ⟹ SISTEMA È STABILE
- |Gloop(jω)| Taglia l'asse 0dB con pendenza -60dB/dec o superiore ⟹ SISTEMA È INSTABILE
- |Gloop(jω)| Taglia l'asse 0dB con pendenza -40dB ⟹ ARRANGIATI !!

ESEMPIO

GUADAGNO AD ANELLO APERTO DELL'OPAMP :

$-40\,dB \Rightarrow$ ARRANGIATI ...

ESEMPIO



GUADAGNO AD ANELLO APERTO DELL'OPAMP:

$$A(s) = \frac{A_0}{(1+s\tau_1)(1+s\tau_2)}$$

$$f_{p_1} = \frac{1}{2\pi\tau_1} = 500\,kHz$$

$$f_{p_2} = \frac{1}{2\pi\tau_2} = 20\,MHz$$

$$A_0 = 80\,dB = 10^4$$

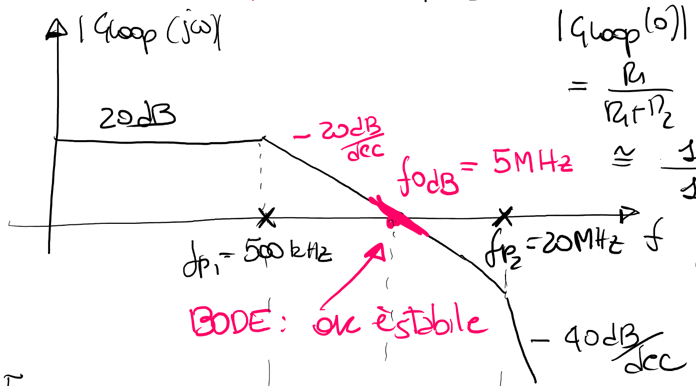
(A) $R_2 = 100\,k\Omega$, $R_1 = 100\,\Omega$

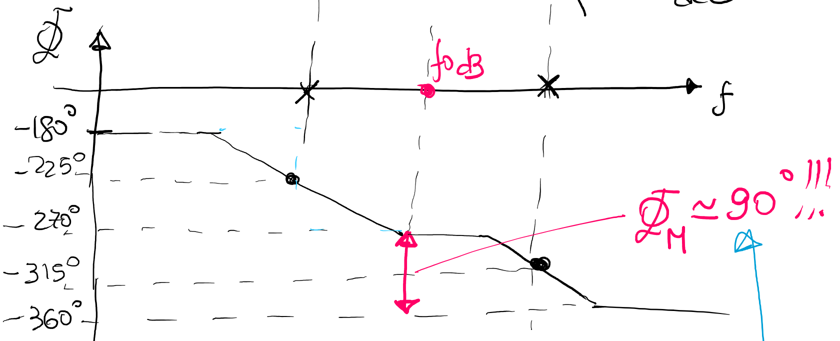
$$G_{ideale} = 1 + \frac{R_2}{R_1} = 1 + \frac{10^5}{10^2} \simeq 1000$$

CALCOLARE IL MARGINE DI FASE.

$$G_{loop}(s) = \frac{v_{loop}(s)}{v_p(s)} = -\frac{R_1}{R_1+R_2} A(s)$$

$$|G_{loop}(0)| = \frac{R_1}{R_1+R_2} A_0 \cong \frac{10^2}{10^5} \cdot 10^4 = 10 \to 20\,dB$$





MARGINE DI FASE

$$\Phi_M = \left[-180° - \operatorname{arctg}\frac{f_{0dB}}{f_{p_1}} - \operatorname{arctg}\frac{f_{0dB}}{f_{p_2}}\right] - (-360°) =$$

$$\simeq -180° - 84° - 14° + 360° = 82°$$

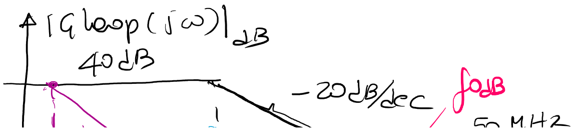
(B) Se abbassassimo R_2 a $10\,k\Omega$?

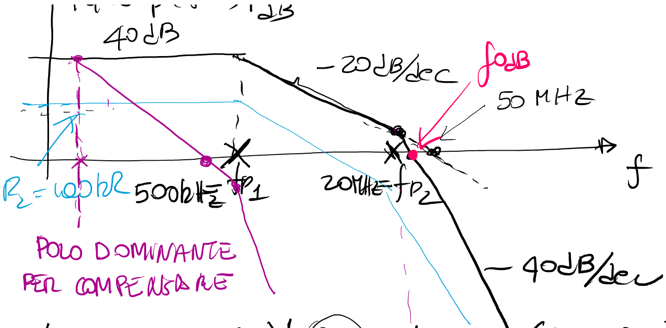
$$G_{ideale} = 1 + \frac{R_2}{R_1} = 101 \Rightarrow \simeq 40\,dB$$

$$G_{loop}(s) = -\frac{R_1}{R_1+R_2} A(s) = -\frac{R_1}{R_1+R_2} \cdot \frac{A_0}{(1+s\tau_1)(1+s\tau_2)} =$$

$$= \frac{10^2}{10^4+10^2} \cdot \frac{10^4}{(1+s\tau_1)(1+s\tau_2)}$$

$$G_{loop}(0) = 100 \Rightarrow 40\,dB$$





$$|G_{loop}(j2\pi f_{P2})|\, f_{P2} = |G_{loop}(j2\pi f_{P1})|\, f_{P1}$$

$$f_{P2} = f_{P1} \cdot \frac{|G_{loop}(j2\pi f_{P1})|}{|G_{loop}(j2\pi f_{P2})|}$$

$$|G_{loop}(j2\pi f_{P2})|\, f_{P2}^2 = f_{0dB}^2 * 1$$

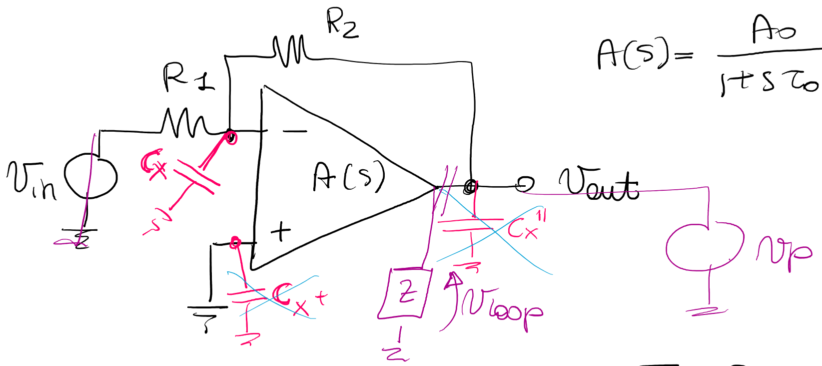
$$f_{0dB} = \sqrt{|G_{loop}(j2\pi f_{P2})|\, f_P^2} = 31.7\,\text{MHz}$$

$$\Phi_M = \left[180^\circ - \arctan\frac{f_{0dB}}{f_{P1}} - \arctan\frac{f_{0dB}}{f_{P2}}\right] - (-360^\circ) =$$

$$= -180^\circ - 89^\circ - 58^\circ + 360^\circ = 33^\circ \;!!!$$

$\Phi_M < 45^\circ \Rightarrow$ il sistema è stabile da un punto di vista matematico MA !!!

EFFETTO DI UN CARICO CAPACITIVO IN INGRESSO



$$A(s) = \frac{A_0}{1 + s\tau_0}$$

C_x capacità parassita al morsetto $\ominus$

$$G_{ideale} = -\frac{R_2}{R_1} \quad \text{grazie alla terra virtuale}$$

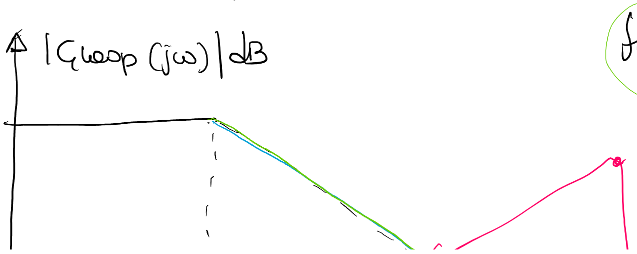
Calcoliamo $G_{loop}(s)$:

$$G_{loop}(s) = \underbrace{-\frac{R_1}{R_1+R_2} A_0}_{G_{loop}(0)} \cdot \frac{1}{1+s\tau_0} \cdot \frac{1}{1+s\tau_p}$$

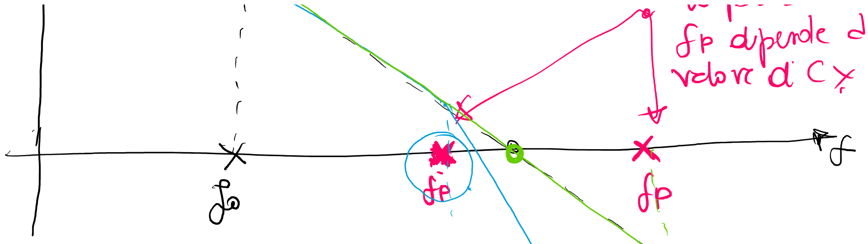
$$\tau_p = C_x (R_1 /\!/ R_2)$$

$$f_0 = \frac{1}{2\pi\tau_0}$$

$$f_p = \frac{1}{2\pi\tau_p}$$



la posizione di f_p dipende dal valore di C_x

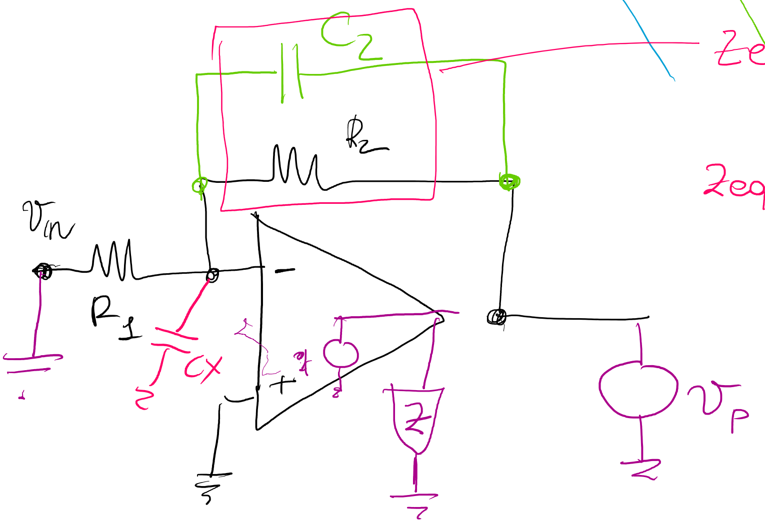


f_p dipende dal valore di C_x

$$Z_{eq}(s) = \frac{R_2}{1 + sC_2R_2}$$

$$Z_{eq}(s) \to \infty \qquad s = -\frac{1}{C_2R_2}$$

$$\tau_2 = C_2R_2$$

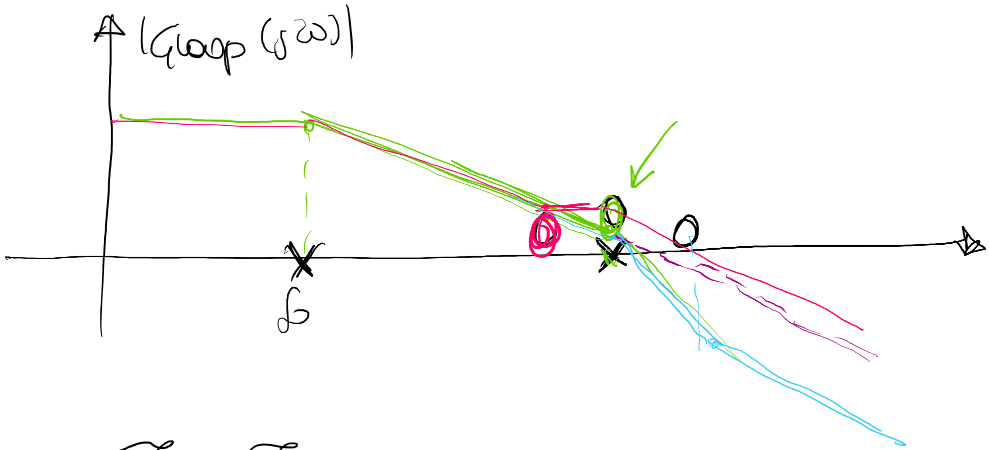


$$G_{loop}(s) = G_{loop}(0)\,\frac{1}{s\tau_0 + 1}\cdot\frac{1 + s\tau_z}{1 + s\tau_p}$$

$$\tau_p = (C_x + C_2)(R_1 /\!/ R_2)$$

COMPENSAZIONE POLO – ZERO

$$\boxed{\tau_z = \tau_p}$$



$$\tau_p = \tau_z$$

$$(C_x + C_2)(R_1 /\!/ R_2) = C_2R_2$$

$$C_x\,\frac{R_1R_2}{R_1+R_2} + C_2\,\frac{R_1R_2}{R_1+R_2} = C_2R_2$$

$$C_x \frac{R_1 R_2}{R_1+R_2} + C_2 \frac{R_1 R_2}{R_1+R_2} = C_2 R_2$$

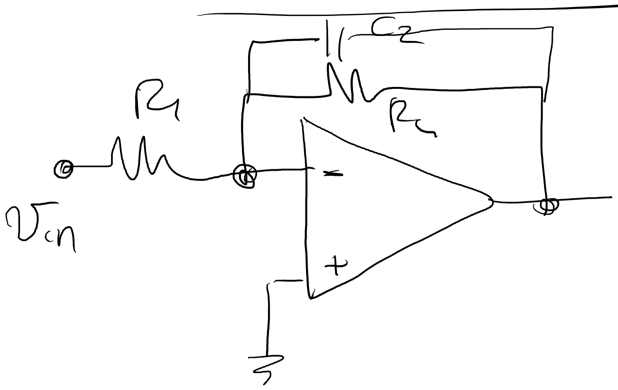
$$C_2 \left[R_2 - \frac{R_1 R_2}{R_1+R_2} \right] = C_x \frac{R_1 R_2}{R_1+R_2}$$

$$C_2 \left[\frac{\cancel{R_2 R_1} + R_2^2 - \cancel{R_1 R_2}}{\cancel{R_1+R_2}} \right] = C_x \frac{R_1 R_2}{\cancel{R_1+R_2}}$$

$$C_2 R_2^{\cancel{2}} = C_x R_1 \cancel{R_2}$$

$$\boxed{C_2 \geq C_x \frac{R_1}{R_2}}$$

valore non noto



$$G_{ideale}(s) = - \frac{Z_2(s)}{Z_1(s)} = - \frac{\frac{R_2}{1+sC_2R_2}}{R_1} =$$

$$= - \frac{R_2}{R_1} \frac{1}{1+sC_2R_2}$$

$$f_p = 1/2\pi C_2 R_2$$