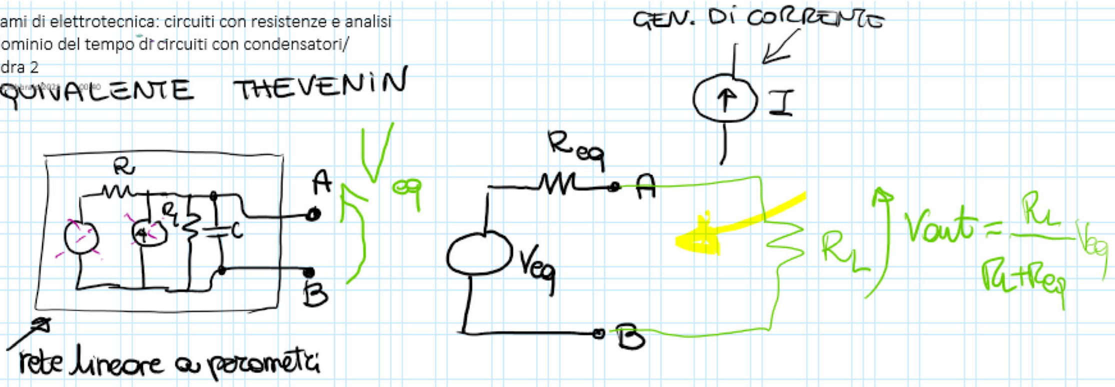
Richiami di elettrotecnica: circuiti con resistenze e analisi nel dominio del tempo di circuiti con condensatori/
Squadra 2

## EQUIVALENTE THEVENIN



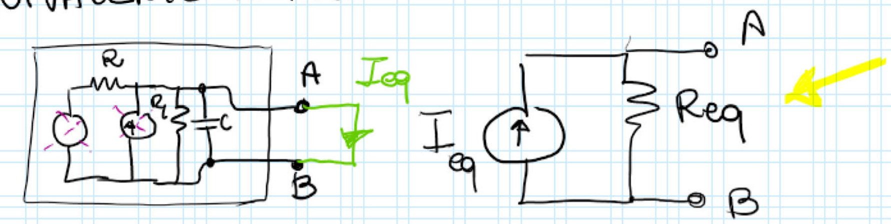

rete lineare a parametri concentrati

$R_{eq}$ 1. Spengo gen. forzanti
2. applico gen. di tensione (o corrente) e valuto la corrente che lo attraversa (o la tensione ai suoi capi)

$$R_{eq} \triangleq \frac{V_P}{I_P}$$

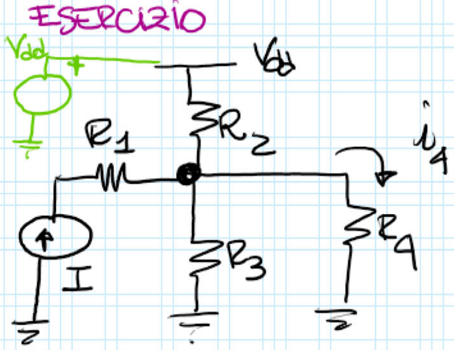
$V_{eq}$: Tensione a vuoto ai morsetti A e B

## EQUIVALENTE NORTON




$$R_{eq} = \frac{V_{eq}}{I_{eq}}$$

**ESERCIZIO**

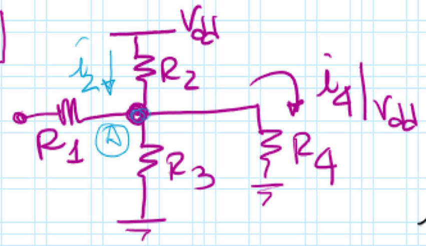



$V_{dd} = +5V$ $\quad I = 2mA$
$R_1 = 400 k\Omega$ $\quad R_4 = 10 k\Omega$
$R_2 = 500 k\Omega$
$R_3 = 100 k\Omega$

1. calcolare la corrente $i_4$
2. si supponga di misurare la corrente $i_4$ con un amperometro reale ($R_{int} = 500\,\Omega$). Quale è la corrente misurata? Quale la caduta di tensione ai capi dell'amperometro?

1.

$V_{dd}$

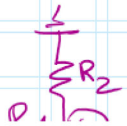



$$i_2 = \frac{V_{dd}}{R_2 + R_3 \parallel R_4}$$

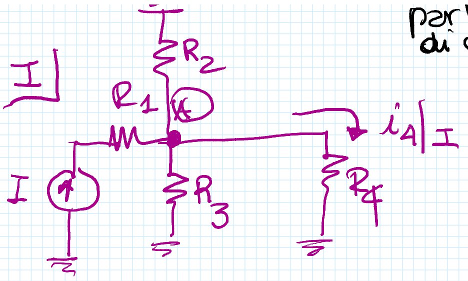
$$i_4|_{V_{dd}} = \frac{R_3}{R_3 + R_4} i_2 = 8.93 \mu A$$

partitore di corrente al nodo A

I

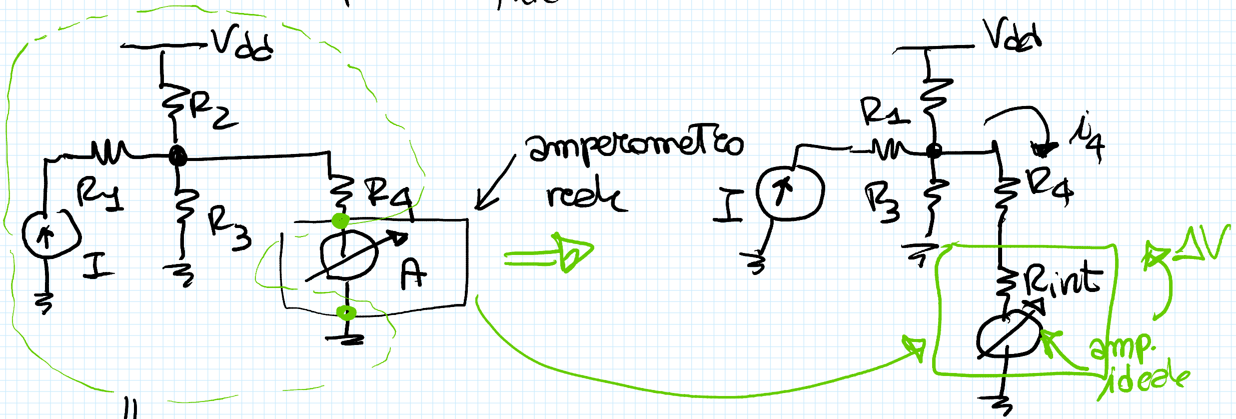


partitore di corrente al nodo (A)

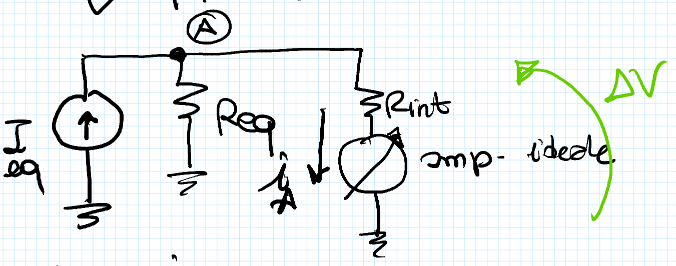



$$i_4\big|_I = I\,\frac{R_2 \parallel R_3}{R_4 + R_2 \parallel R_3} = 1.8\,mA$$

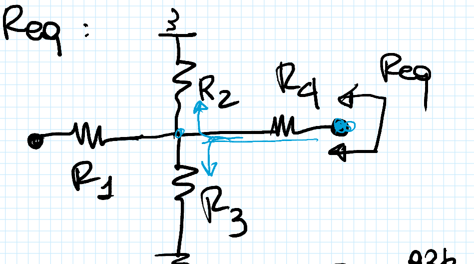
$$i_4 = i_4\big|_{V_{dd}} + i_4\big|_I \simeq 1.8\ mA$$




⇓ eq. Norton




Req :




$$R_{eq} = R_4 + R_2 \parallel R_3 = 93k$$

$$I_{eq} = i_4$$

↑ calcolata precedentemente

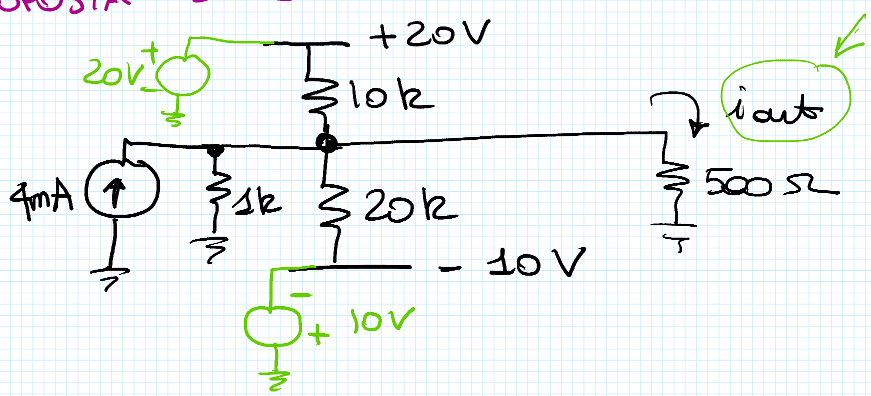
partitore di corrente

$$i_A = I_{eq}\,\frac{R_{eq}}{R_{eq} + R_{int}} = 1.79\,mA$$

↑ corrente misurata dall'amperometro

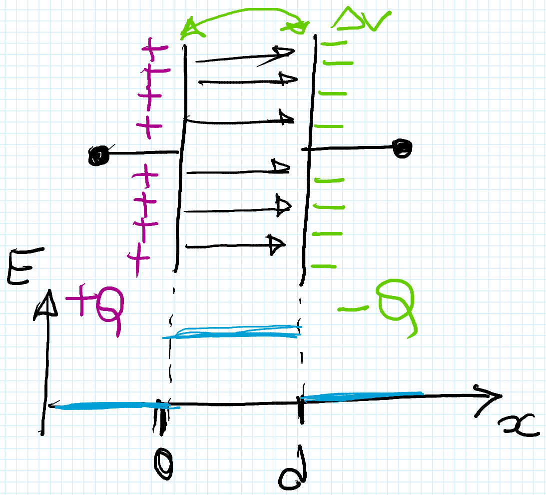
$$\Delta V = i_A * R_{int} = 0.89V \quad !!!$$

## PROPOSTA DI ESERCIZIO




# RETI ELETTRICHE NEL DOMINIO DEL TEMPO

## CONDENSATORE




Teorema di Gauss

$$\int_{\Sigma} \vec{E} \cdot \vec{n}\, d\sigma = \int_{\Omega} \frac{\rho}{\varepsilon}\, dV$$

densità superficiale di carica

$$A\,E = \frac{Q}{\varepsilon} \Rightarrow E = \frac{Q}{A\,\varepsilon} = \frac{\sigma}{\varepsilon}$$

campo elettrico

$$\Delta V = E \cdot d = \frac{Q\,d}{A\,\varepsilon} = \frac{Q}{C}$$

CAPACITÀ

$$C = \frac{A \cdot \varepsilon}{d}$$

[FARAD]

$1\,nF = 10^{-9}\,F$

$1\,pF = 10^{-12}\,F$

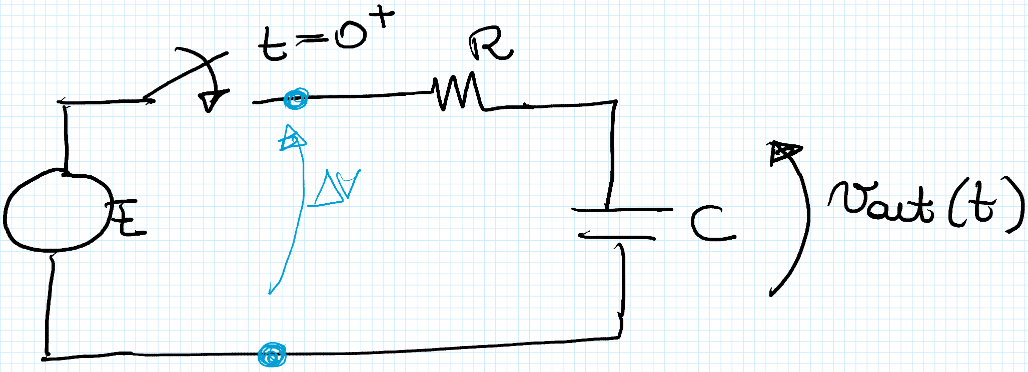
$1\,fF = 10^{-15}\,F$

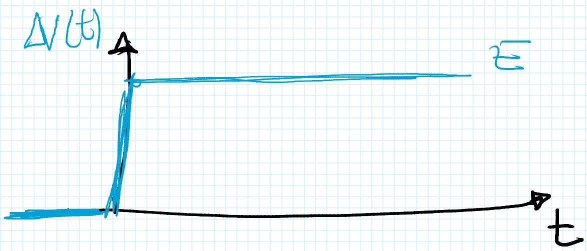
$$dQ = i(t)\,dt \qquad dV(t) = \frac{dQ}{C} = \frac{i(t)\,dt}{C}$$

$$\boxed{i(t) = C\,\frac{dV(t)}{dt}}$$

RELAZIONE COSTITUTIVA DEL CONDENSATORE

## CIRCUITO RC (SINGLE TIME CONSTANT CIRCUITS STC CIRCUITS



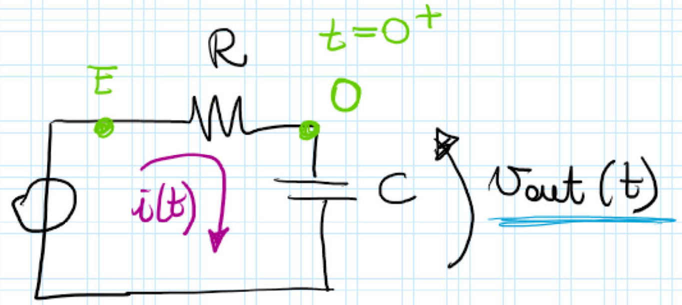

GRADINO DI TENSIONE

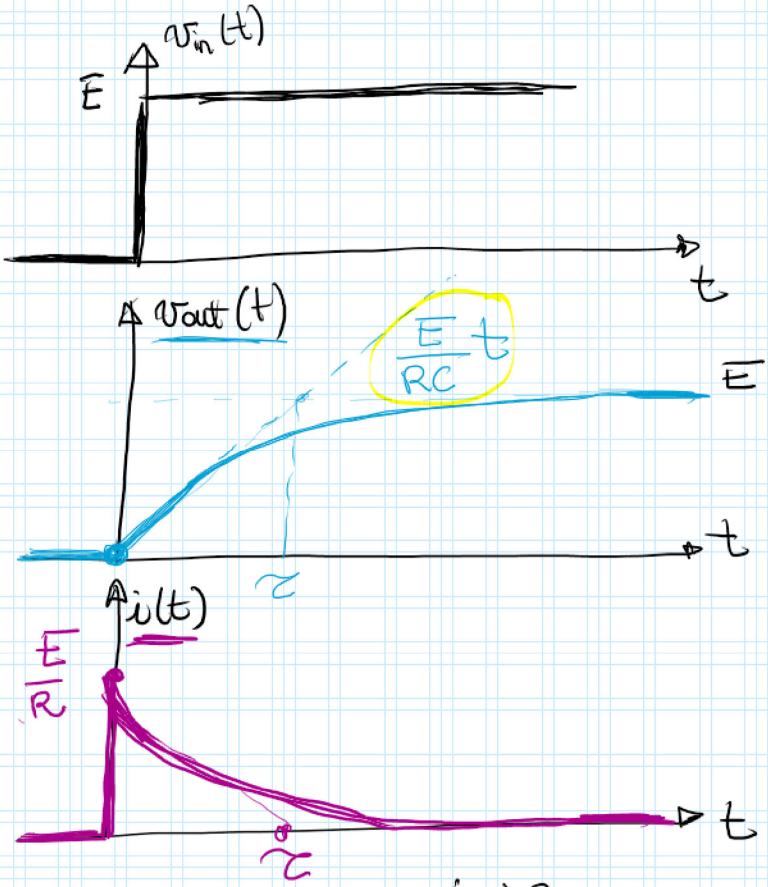
VOLTAGE STEP

$$1(t) = \begin{cases} 0 & t \leq 0^- \\ 1 & t \geq 0^+ \end{cases}$$

$\Downarrow$

$v_{in}(t) =$
$= E \cdot 1(t)$




t piccoli

$$I = \frac{E}{R} = \frac{Q}{t}$$

$$Q = \frac{E}{R} t$$

$$v_{out}(t) = \frac{Q(t)}{C} = \frac{E}{RC} t$$

$$i(t) = C \frac{d v_{out}}{dt}$$

$$v_{out}(t) = E - i(t) R$$

$$\boxed{v_{out}(t) = E - CR \frac{d v_{out}(t)}{dt}}$$

$\Downarrow$

$$v_{out}(t) = E \left[ 1 - \exp\left( - \frac{t}{RC} \right) \right] \simeq \frac{E}{RC} t \qquad t \ll \tau$$

$\tau = RC$ cost. di tempo

METODO DI ANALISI DEI CIRCUITI A SINGOLA COSTANTE DI TEMPO

1) Calcolo $\tau$ : * spengo i gen. forzanti

① Calcolo $\tau$ :

- ✱ spengo i gen. forzanti
- ✱ Trovo la resistenza equivalente in parallelo al condensatore
  - • un solo C
    - – tolgo C
    - – valuto Req
  - • (una sola R) tanti condensatori d.p
    - – tolgo R
    - – valuto Ceq
  - • tanti condensatori dipendenti / tante resistenze
    - ridurre la rete

      ↓

      Req   Ceq

$$\tau = R_{eq} C_{eq}$$

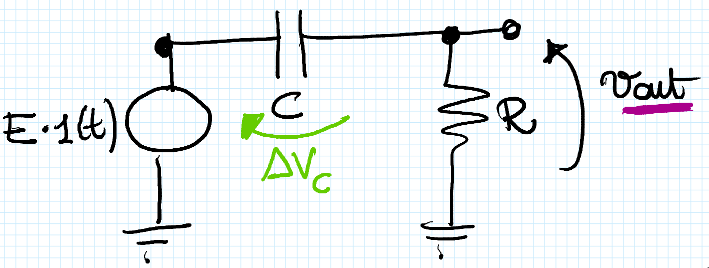
② Calcolo il valore della variabile di uscita a regime per $t \to \infty$ (il condensatore si comporta da circuito aperto)

③ Calcolo il valore della variabile di uscita per $t = 0^+$

$$\Delta V_C(0^+) = \Delta V_C(0^-)$$

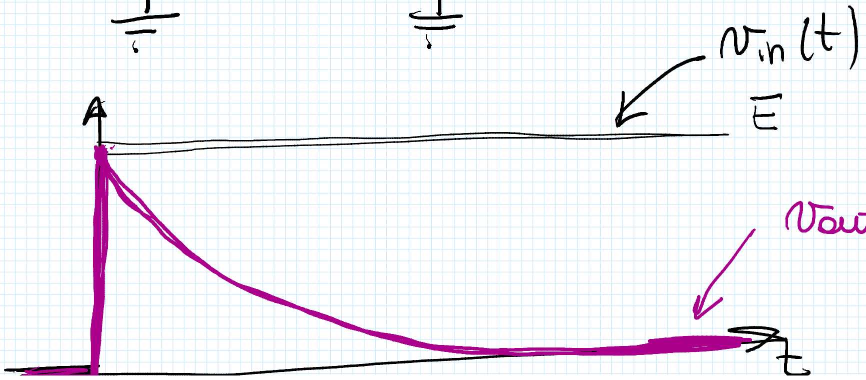
④ Raccordo i valori iniziale e finale con andamento exp a singola costante di tempo ($\tau$)

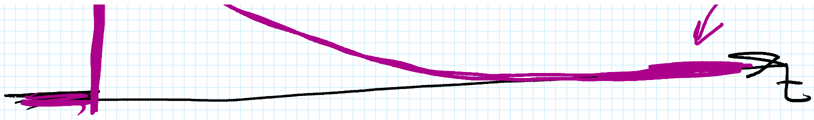
CIRCUITO CR




$\tau = RC$

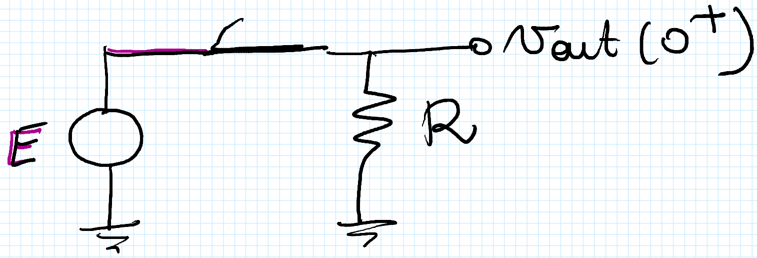
$t \to \infty$   $v_{out}(t \to \infty) \to 0$



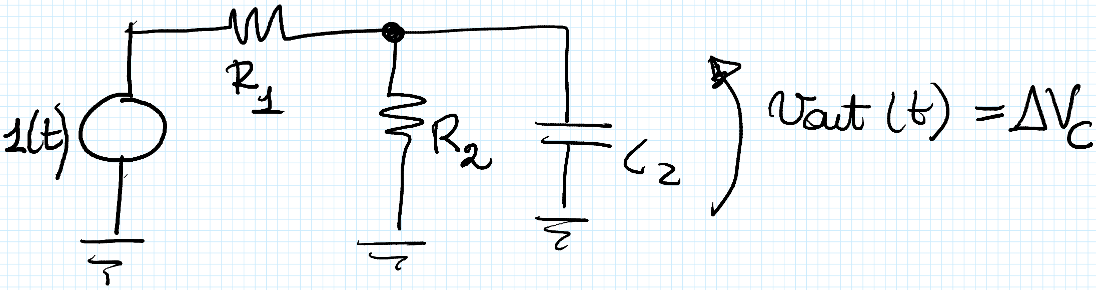



$t = 0^-$ $\quad \Delta V_C(0^-) = 0\,V$

$t = 0^+$ $\quad \Delta V_C(0^+) = \Delta V_C(0^-)$

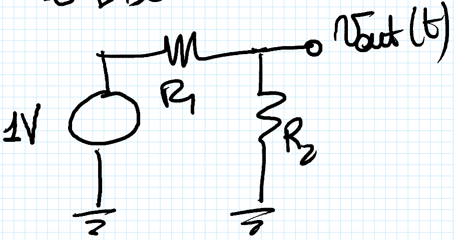


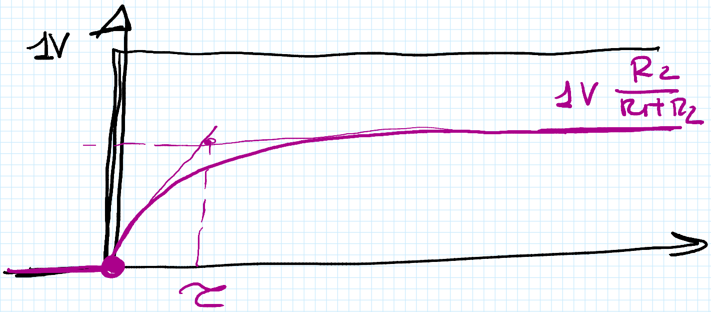

ESERCIZIO




$\tau = C_2\,(R_1 \| R_2)$

$t \to \infty$

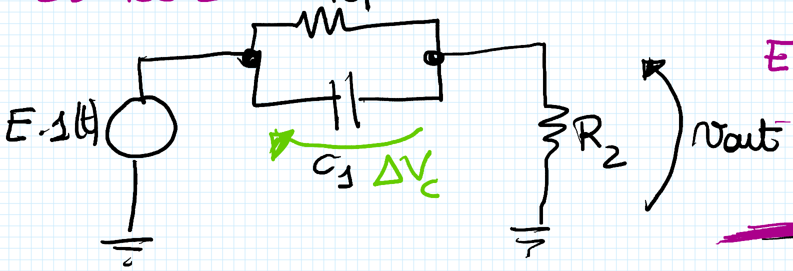


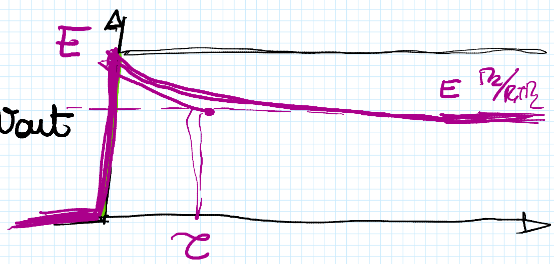




$$v_{out}(t \to \infty) = 1V\,\frac{R_2}{R_1 + R_2}$$

$t = 0^+$ $\quad \Delta V_C(0^+) = \Delta V_C(0^-) = 0$

ESERCIZIO



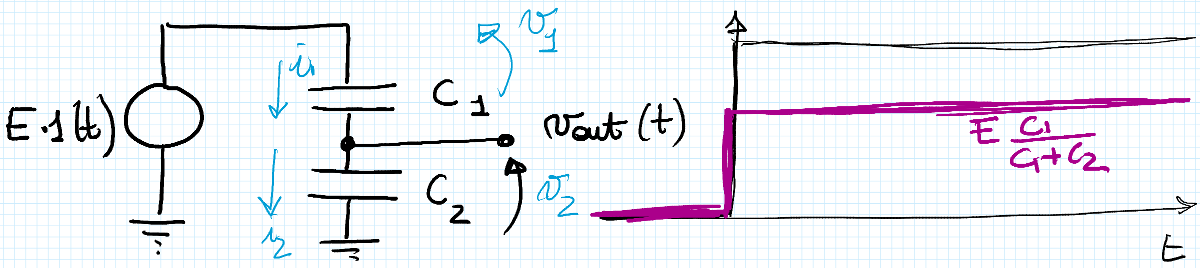



$$\tau = C\,(R_1 \parallel R_2)$$

$t \to \infty \qquad v_{out}(t\to\infty) = \dfrac{R_2}{R_1+R_2}\,E$

$t = 0^+ \qquad \Delta V_c(0^+) = \Delta V_c(0^-) = 0 \qquad v_{out}(0^+) = E$

PARTITORE CAPACITIVO




$i_1 = i_2$

$$C_1 \frac{dv_1}{dt} = C_2 \frac{dv_2}{dt}$$

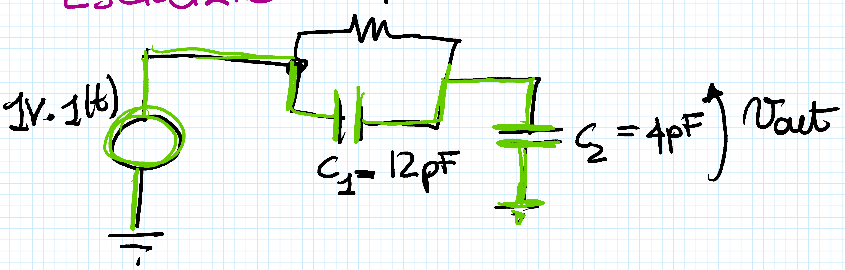
$$\int dv_2 = \frac{C_1}{C_2}\int dv_1$$

$$v_2 = \frac{C_1}{C_2}\,v_1 = \frac{C_1}{C_2}\left[E - v_2\right] = E\,\frac{C_1}{C_2} - \frac{C_1}{C_2}\,v_2$$

$$v_2\left(1+\frac{C_1}{C_2}\right) = \frac{C_1}{C_2}\,E$$

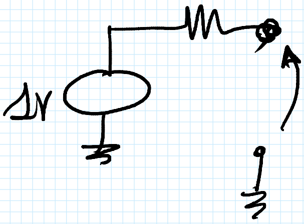
$$v_{out} = v_2 = \frac{C_1/C_2}{1+C_1/C_2}\,E = \frac{C_1}{C_1+C_2}\,E$$

ESERCIZIO




$\tau = R_1\,(C_1+C_2) =$
$= 16\,ns$


$t \to \infty$




$v_{out}(t \to \infty) = 1V$

$t = 0^+$

$$v_{out}(0^+) = 1V \frac{C_1}{C_1 + C_2} = 1V \frac{12}{16} = 0.75V$$

PARTITORE COMPENSATO