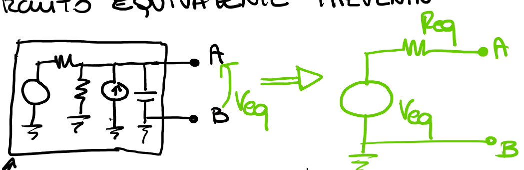
Richiami di elettrotecnica: circuiti con resistenze e analisi nel dominio del tempo di circuiti a singola costante di tempo / Squadra 1

Wednesday, February 24, 2021 10:16 AM

gen. di corrente

CIRCUITO EQUIVALENTE THEVENIN



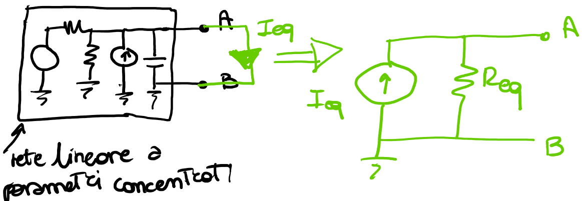
rete lineare a parametri concentrati

R_{eq}: • spegnere i gen. forzanti
• applicare un gen di tensione (o di corrente) e valutare la corrente che lo attraversa (oppure la tensione ai suoi capi)

$$R_{eq} \triangleq \frac{v_p}{i_p}$$

V_{eq}: tensione a vuoto ai morsetti

CIRCUITO EQUIVALENTE NORTON

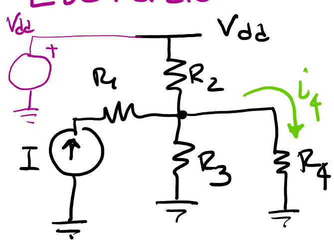


rete lineare a parametri concentrati

I_{eq}: corrente di cortocircuito ai morsetti

$$R_{eq} = \frac{V_{eq}}{I_{eq}}$$

ESERCIZIO



$V_{dd} = +5V$

$R_1 = 400 k\Omega$

$R_2 = 500 k\Omega$

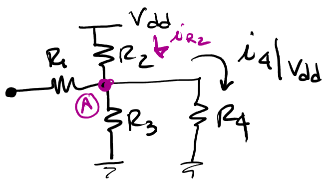
$R_3 = 100 k\Omega$

$R_4 = 10 k\Omega$

$I = 2mA$

1. corrente in R_4, i_4?
2. amperometro reale ($R_{int} = 500\,\Omega$) per la misura di i_4, quale il valore di corrente misurato? Quale è la caduta di tensione ai capi dell'amperometro?

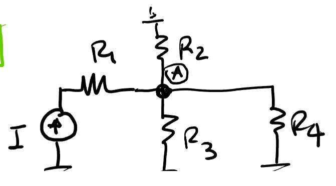
$V_{dd}|$

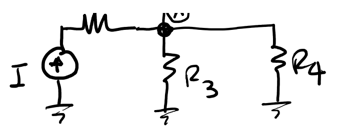


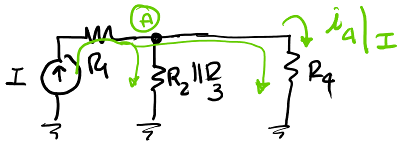
$$i_{R_2} = \frac{V_{dd}}{R_2 + R_3 /\!/ R_4} = 9.8\,\mu A$$

$$i_4|_{V_{dd}} = \frac{R_3}{R_3 + R_4}\, i_{R_2} = 8.9\,\mu A$$

$I|$





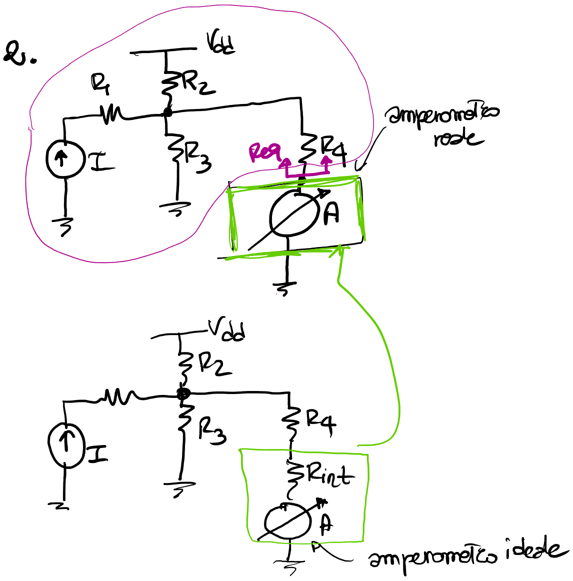


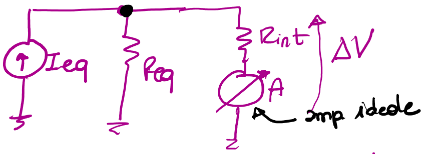
$$i_4\big|_I = I\,\frac{R_2\|R_3}{R_4 + R_2\|R_3} = 1.8\,mA$$

Per il princ. di sovrapposizione degli effetti:

$$i_4 = i_4\big|_{V_{dd}} + i_4\big|_I \cong 1.8\,mA$$

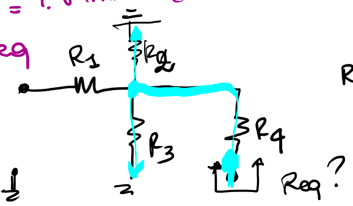
2.





$I_{eq} = 1.8\,mA$ (calcolata al punto 1)

Req

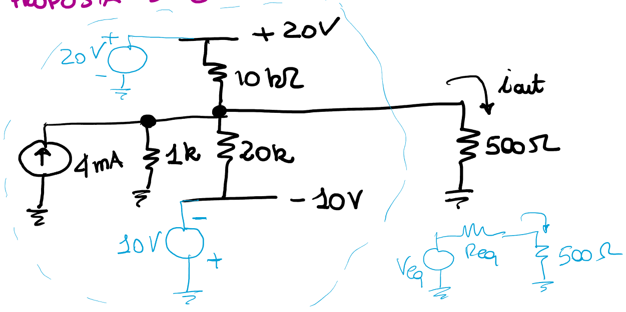


$$R_{eq} = R_4 + R_2\|R_3 = 93\,k\Omega$$

$$i_4\big|_{meas} = I_{eq}\,\frac{R_{eq}}{R_{eq} + R_{int}} = 1.79\,mA$$

$$\Delta V = i_4\big|_{meas} * R_{int}$$

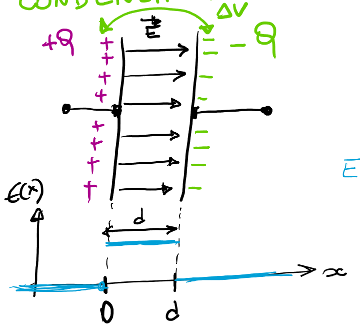
PROPOSTA DI ESERCIZIO



RETI ELETTRICHE NEL DOMINIO DEL

RETI ELETTRICHE NEL DOMINIO DEL TEMPO

CONDENSATORE



Teorema di Gauss

$$\int_{\Sigma'} \vec{E} \cdot \vec{n} \, d\sigma = \int_{\Omega} \frac{\rho}{\varepsilon} dV$$

$$E \cdot A = \frac{Q}{\varepsilon} \Rightarrow E = \frac{Q}{A\varepsilon} = \frac{\sigma}{\varepsilon}$$

densità superficiale di carica

$$\Delta V = E \cdot d = \frac{Q d}{A \varepsilon} = \frac{Q}{C}$$

$$\boxed{C = \frac{A \, \varepsilon}{d}}$$ CAPACITÀ [FARAD]

$10^{-6} \to \mu$

$10^{-9} \to$ nano

$10^{-12} \to$ pico

$10^{-15} \to$ femto

Se Q varia nel tempo

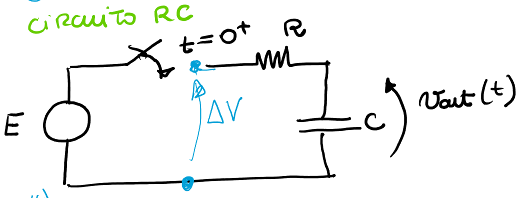
$$dV(t) = \frac{dQ(t)}{C} = \frac{i(t)\,dt}{C}$$

$$dQ = i(t)\,dt$$

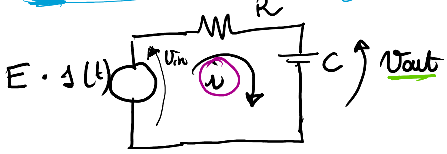
$$\boxed{i(t) = C \frac{dv(t)}{dt}}$$ RELAZIONE COSTITUTIVA DEL CONDENSATORE

CIRCUITI A SINGOLA COSTANTE DI TEMPO

CIRCUITO RC

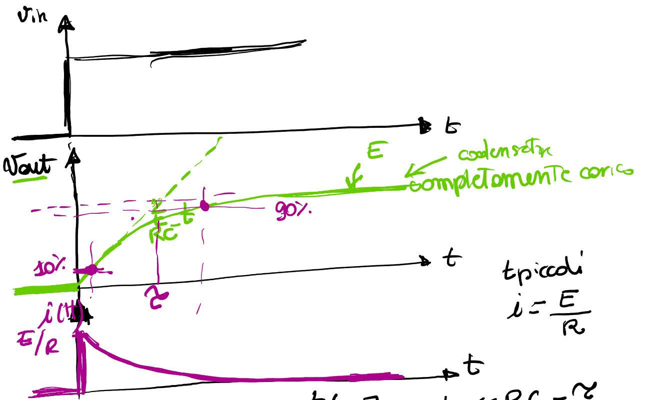


$$1(t) = \begin{cases} 0 & t < 0^- \\ 1 & t \geq 0^+ \end{cases}$$



$$i(t) = C \frac{dv_c}{dt}$$

$$v_{out}(t) = E - i(t)R = E - RC \frac{dv_{out}}{dt}$$



tipicamente $i = \frac{E}{R}$

$= \left[1 - e^{-t/RC}\right]$

$t \ll RC = \tau$

cost. di tempo

$$V_{out} = E\left[1 - e^{-t/RC}\right] \qquad t \ll RC = \tau$$

$\uparrow$ cost. di Tempo

$\bar{t} \to 0 \qquad V_{out}(t) = \frac{E}{RC}\,t$ per t piccoli

$t_{10-90\%}$: tempo di salita

$t_{10-90\%} = 2.2\,\tau \qquad \tau$: proprietà topologica della rete

METODO DI ANALISI DEI CIRCUITI A SINGOLA COSTANTE DI TEMPO

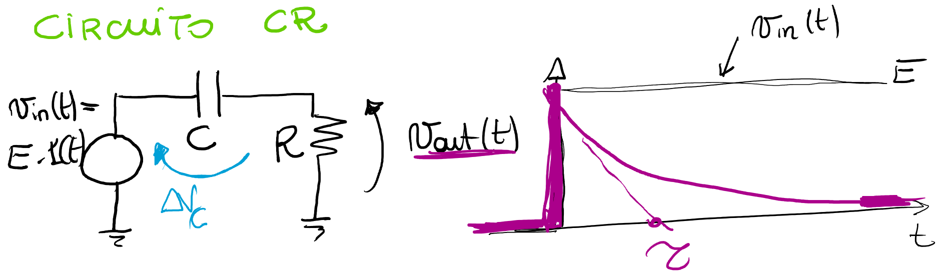
1. Calcolare la costante di Tempo, τ
   a. spengo i generatori forzanti
   b. calcolo la resistenza equivalente in parallelo ai morsetti del condensatore (o il condensatore equivalente in parallelo alla resistenza)
      - un condensatore solo
        - rimuovo C e calcolo R_{eq} ai morsetti
      - più condensatori tra loro dipendenti
        - rimuovere R e calcolo C_{eq}
      - più condensatori dipendenti e più resistenze $\Rightarrow$ semplifico la rete
2. Calcolo il valore della variabile di uscita per $t \to \infty$ (il condensatore si comporta da circuito aperto)
3. Calcolare l'andamento sul fronte $(t = 0^+)$

$$\Delta V_C(0^+) = \Delta V_C(0^-)$$

(il condensatore sul fronte si comporta come un generatore di Tensione pari a $\Delta V_C(0^-)$)

4. Raccordo con andamento esponenziale a singola τ il valore iniziale e finale

CIRCUITO CR

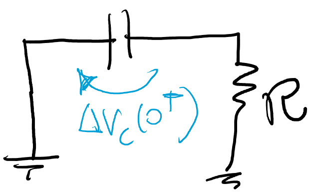


$$\tau = RC$$

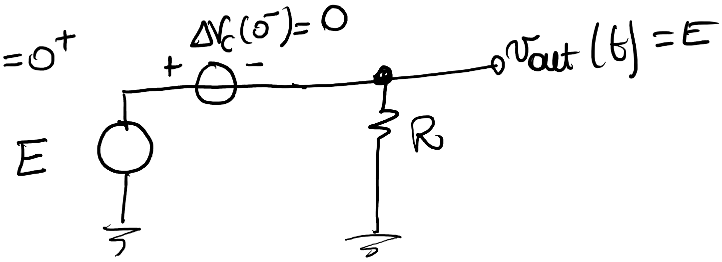
- $t \to \infty$ $\quad v_{out}(t \to \infty) \to 0$
- $t = 0^+$

$$\Delta V_c(0^+) = \Delta V_c(0^-) \cong 0$$

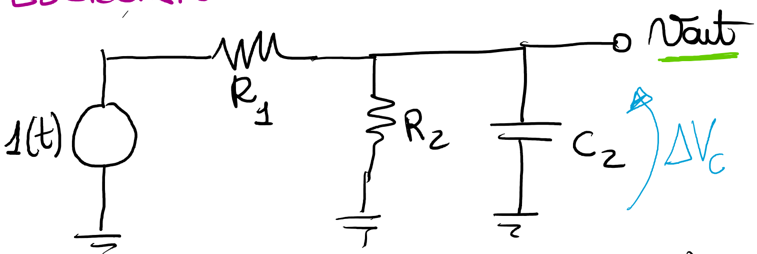
$t = 0^-$

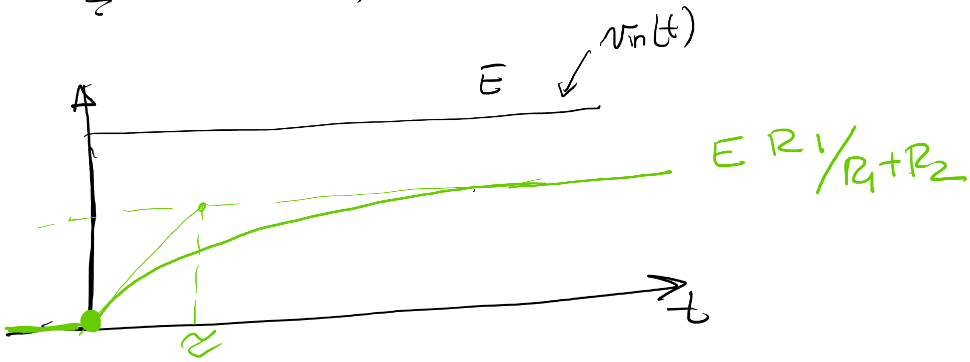


$t = 0^+$

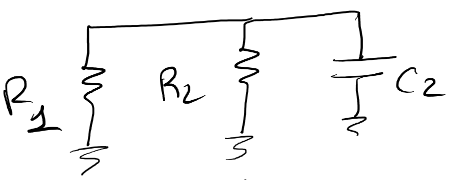


ESERCIZIO

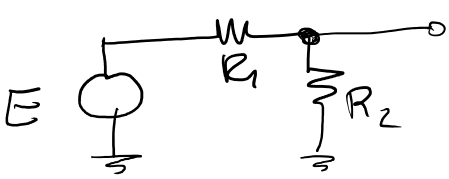




$$\tau = C_2 (R_1 \| R_2)$$



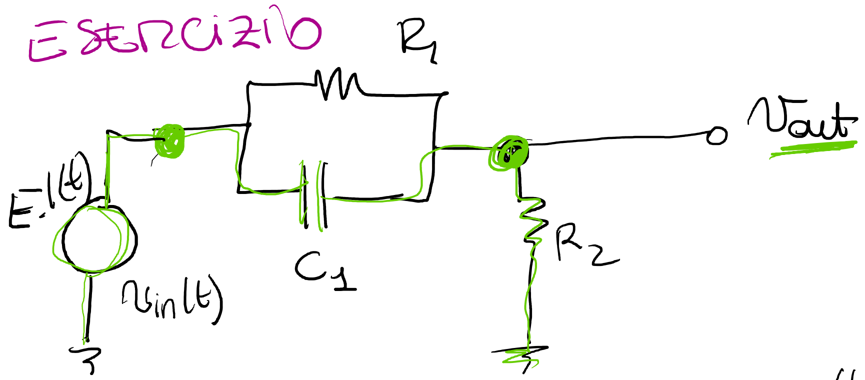
$v_{out}(t \to \infty)$

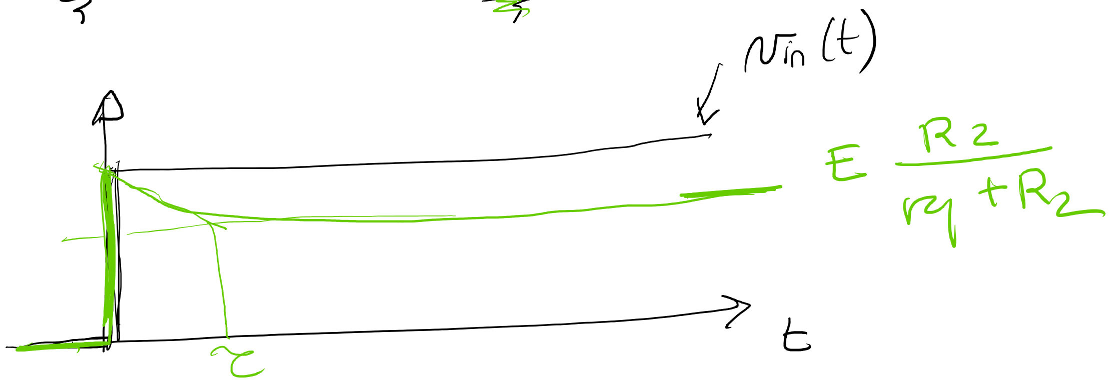


$$v_{out}(t \to \infty) = \frac{R_2}{R_1 + R_2} E$$

$t = 0^+$ $\quad \Delta V_c(0^+) = \Delta V_c(0^-) = 0$

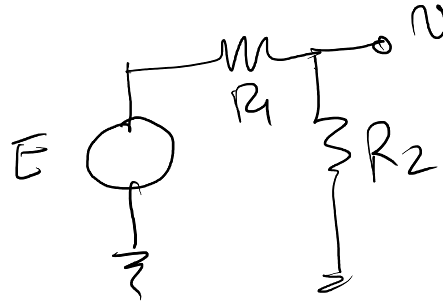
ESERCIZIO





* $\tau = C_1 (R_1 \parallel R_2)$

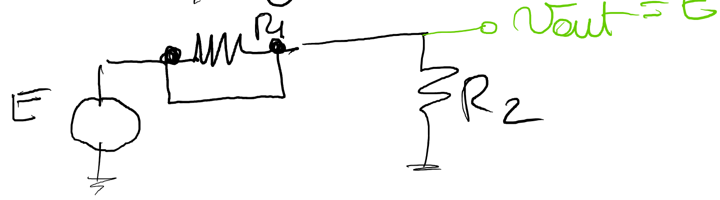
* $t \to \infty$

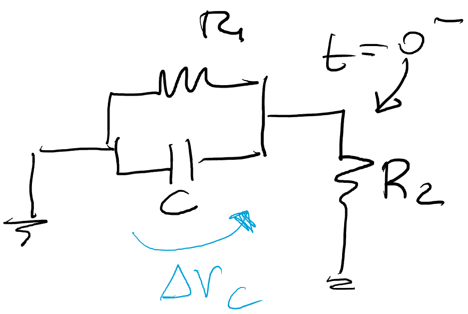


$$V_{out} = \frac{R_2}{R_1 + R_2} E$$

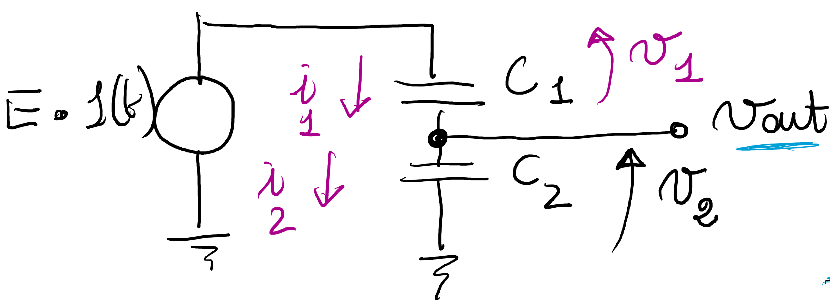
* $t = 0^+$

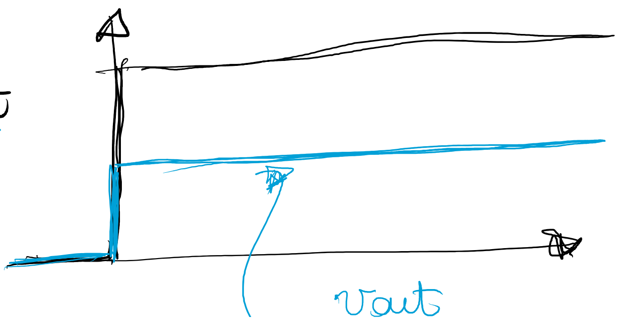
$$\Delta V_C(0^+) = \Delta V_C(0^-) = 0$$

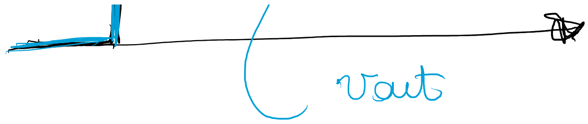




PARTITORE CAPACITIVO







$$i_1 = i_2$$

$$C_1 \frac{dv_1}{dt} = C_2 \frac{dv_2}{dt}$$

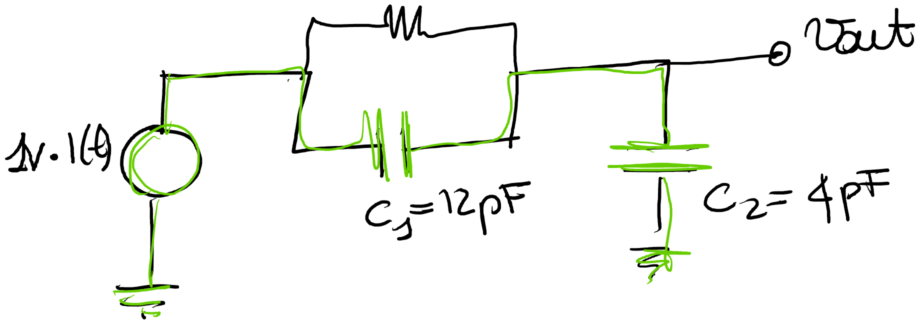
$$\frac{C_1}{C_2} = \frac{dv_2}{dv_1} \qquad dv_2 = \frac{C_1}{C_2} dv_1$$

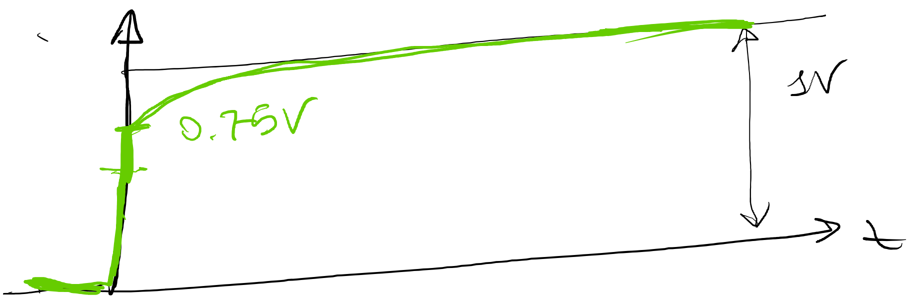
$$v_{out} = v_2 = \frac{C_1}{C_2} v_1 = \frac{C_1}{C_2}\left[E \cdot 1(t) - v_{out}\right]$$

$$\left(1 + \frac{C_1}{C_2}\right) v_{out} = \frac{C_1}{C_2} E \cdot 1(t)$$

$$v_{out} = \frac{C_1}{\cancel{C_2}} \frac{E \cdot 1(t)\, \cancel{C_2}}{C_1 + C_2} = \frac{C_1}{C_1 + C_2} E \cdot 1(t)$$

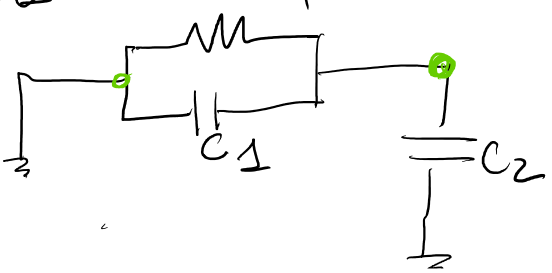
ESERCIZIO $R = 1k\Omega$

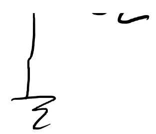




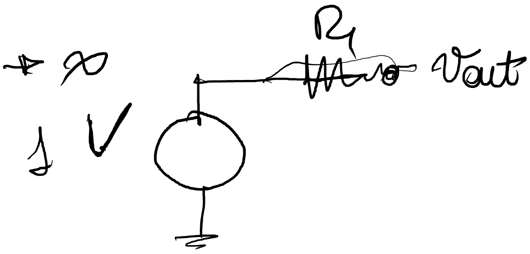
$$\tau = (C_1 + C_2) R = 16ms$$

τ:





$t \to \infty$



$$V_{out}(t=0^+) = 1V \cdot \frac{C_1}{C_1+C_2} = 0.75V$$

PARTITORE COMPENSATO