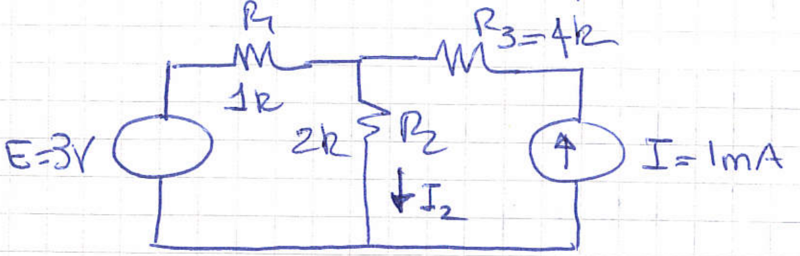
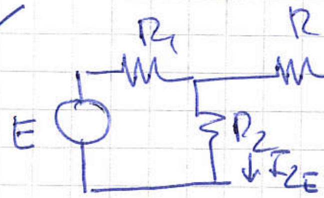
SOVRAPPOSIZIONE DEGLI EFFETTI



Circuito lineare
⇓
sovrapposizione degli effetti

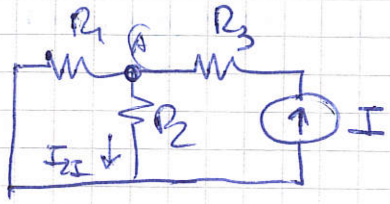
$$I_2 = I_{2_E}' + I_{2_I}$$

E/



$$I_{2_E} = \frac{E}{R_1 + R_2}$$

I



R_3 non conta è in serie a un gen di corrente ideale

partitore di corrente in A

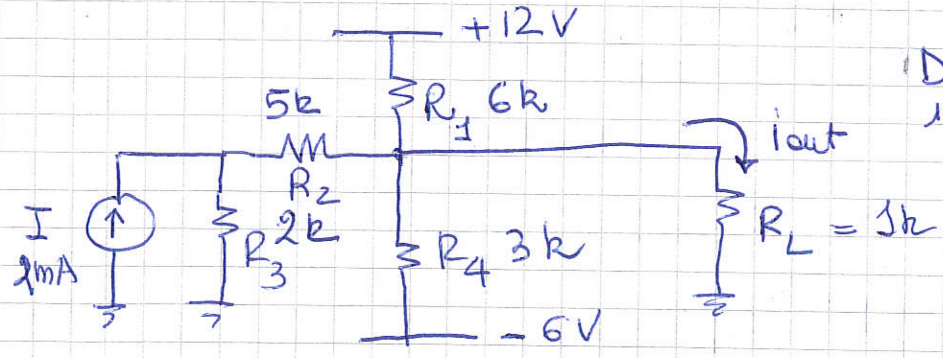
$$I_{2_I} = \frac{R_1}{R_1 + R_2} I$$

⇓

$$I_2 = I_{2_E} + I_{2_I} = \frac{E}{R_1 + R_2} + I \frac{R_1}{R_1 + R_2} =$$

$$= \frac{3V}{3k} + 1mA \frac{1k}{3k} = \frac{4}{3} mA$$

ESERCIZIO

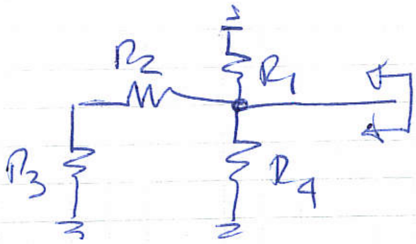


Determinare la corrente i_{out} che fluisce in R_L

- Circuito lineare ⇒ eq. Thevenin e sovrapposizione degli effetti

$$i_{out} = \frac{V_{eq}}{R_{eq} + R_L}$$

• R_{eq}: spengo tutti i gen. forzanti



$$R_{eq} = R_1 \| R_4 \| (R_2 + R_3) =$$
$$= 6k \| 3k \| 7k = \frac{6 \cdot 3k}{6+3} \| 7k =$$
$$= 2k \| 7k = \frac{7 \cdot 2}{7+2} = \frac{14}{9} k = 1.55k$$

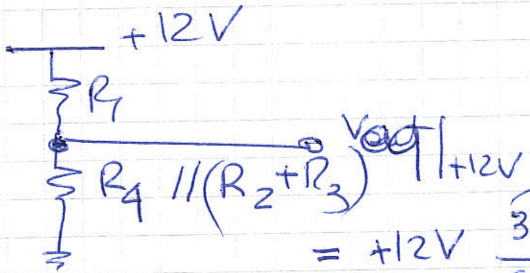
• V_{eq}: sovrapposizione degli effetti, calcolo la tensione a vuoto

$$V_{eq} = V_{eq}|_I + V_{eq}|_{+12V} + V_{eq}|_{-6V}$$

I)

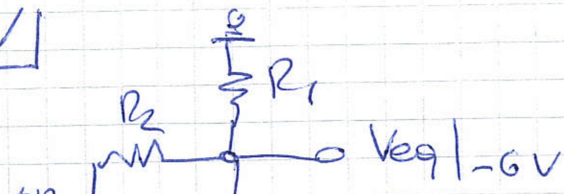
$$V_{eq_I} = \underbrace{I \frac{R_3}{R_3 + [R_2 + R_4 \| R_1]}}_{\text{partitore di corrente}} \cdot R_4 \| R_1 = 2mA \frac{2k}{2k + [5k + \underbrace{3k \| 6k}_{2k}]} \overbrace{(3k \| 6k)}^{2k} =$$
$$= 2mA \; \frac{2}{9} \times 2k = \frac{8}{9} V$$

+12V)



$$V_{eq}|_{+12V} = 12V \frac{R_4 \| (R_2 + R_3)}{R_1 + R_4 \| (R_2 + R_3)} =$$
$$= +12V \frac{\overbrace{3k \| 7k}^{21/10 = 2.1k}}{6k + 3k \| 7k} = +12V \frac{2.1k}{8.1k} =$$
$$= +3.11V$$

-6V)

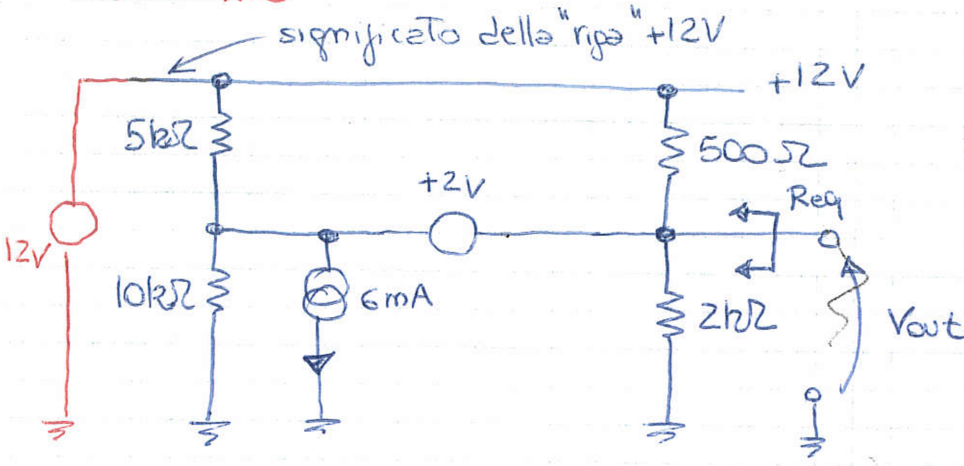


$$V_{eq}|_{-6V} = \frac{R_1 \| (R_2 + R_3)}{R_4 + R_1 \| (R_2 + R_3)} (-6V) =$$
$$= (-6V) \frac{\overbrace{6k \| 7k}^{42/13 \, k}}{3k + 6k \| 7k} = (-6V) \frac{42/13}{3 + 42/13} =$$
$$= -6V \frac{42}{42+39} = (-6V) \frac{42}{81} = -\frac{84}{27} V =$$
$$= -3.11 V$$

$$\Downarrow \; V_{eq} = \frac{8}{9} V + \cancel{3.11V} - \cancel{3.11V} = \frac{8}{9} V = 0.88V$$

$$i_{out} = \frac{8/9 \, V}{\frac{14k}{9} + 1k} = \frac{8V}{(14+9)k} = \frac{8}{23} mA = 0.348 \, mA$$

ESERCIZIO

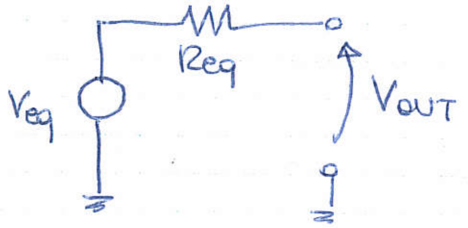


Determinare:

1) Req
2) Vout.
3) Tensione Vout se connetto $R = 1k\Omega$ verso massa in uscita

1) $R_{eq} = 2k\Omega \,//\, 500\Omega \,//\, 5k\Omega \,//\, 10k\Omega = 357\,\Omega$

2) Equivalente Thevenin:



$$V_{eq_1} = 12V \frac{2k // 10k}{(2k // 10k) + (5k // 500)} = 12V \cdot \frac{1.67k}{2.12k} = 9.45V$$

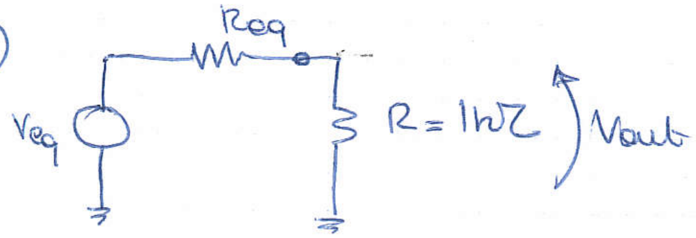
$$V_{eq_2} = -6mA \times R_{eq} = -2.14V$$

$$V_{eq_3} = -2V \frac{2k // 500}{(2k // 500) + (10k // 5k)} = -\frac{0.4k}{3.73k} \times 2V = -0.21V$$

$\Downarrow$

$$V_{eq} = (9.45V - 2.14V - 0.21V) = 7.1V = V_{out}$$

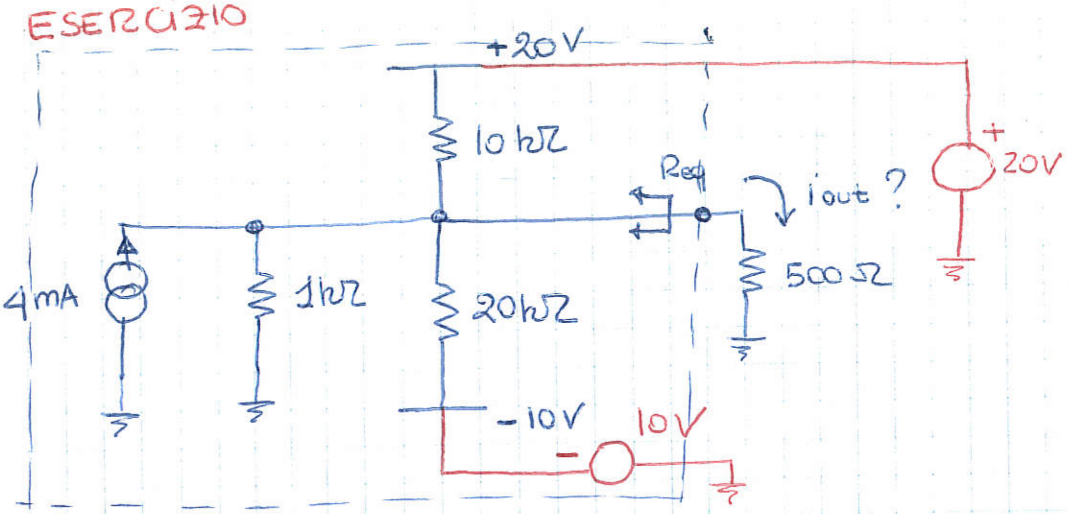
3)



$$V_{out} = \underbrace{\frac{R}{R + R_{eq}}}_{\text{PARTITORE DI TENSIONE}} V_{eq} =$$

$$= \frac{1k}{1k + 357} \, 7.1V = 5.23V$$

ESERCIZIO

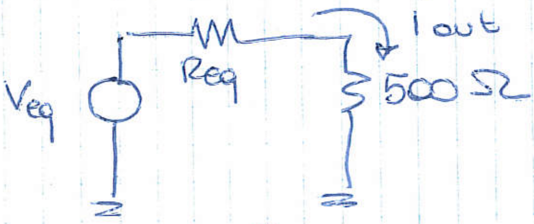


Determinare la corrente di uscita i_{out}.

* significato del +20V e -10V
* -10V: esistono anche tensioni negative! Il generatore di tensione è collegato con il morsetto positivo a massa
* che cosa significa "spegnere" i generatori di tensione e di corrente.

$$R_{eq} = 20k\Omega \parallel 10k\Omega \parallel 1k\Omega = 870\,\Omega$$

Calcoliamo l'EQUIVALENTE THEVENIN DEL CIRCUITO entro la linea tratteggiata:



$$\Rightarrow i_{out} = \frac{V_{eq}}{R_{eq} + 500\,\Omega}$$

$$- V_{eq_1} = \underbrace{\frac{10k \parallel 20k}{1k + (10k \parallel 20k)}}_{\text{PARTITORE DI CORRENTE}} * 4mA * 1k\Omega = \frac{\frac{10*20}{30}k}{\frac{23}{3}k} 1k\, 4mA =$$

$$= \frac{20}{23} * 4mA * 1k\Omega = 3.48V$$

$$- V_{eq_2} = \underbrace{\frac{20k\Omega \parallel 1k\Omega}{10k\Omega + (20k\Omega \parallel 1k\Omega)}}_{\text{PARTITORE DI TENSIONE}} * 20V = \frac{\frac{20}{21}k}{\left(\frac{20}{21} + 10\right)k\Omega} * 20V = 1.74V$$

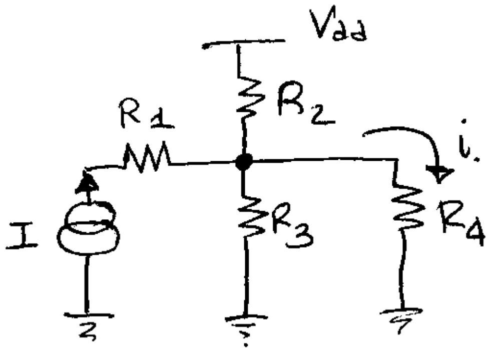
$$- V_{eq_3} = \underbrace{\frac{10k\Omega \parallel 1k\Omega}{20k\Omega + (10k\Omega \parallel 1k\Omega)}}_{\text{PARTITORE DI TENSIONE}} * (-10V) = -0.43V$$

$\Downarrow$

$$V_{eq} = V_{eq_1} + V_{eq_2} + V_{eq_3} = 4.79V \Rightarrow i_{out} = \frac{V_{eq}}{R_{eq} + 500\,\Omega} = 3.5\,mA$$

EXERCISE

Let's consider The following circuit:



$V_{dd} = +5V$ $\quad$ $I = 2\,mA$

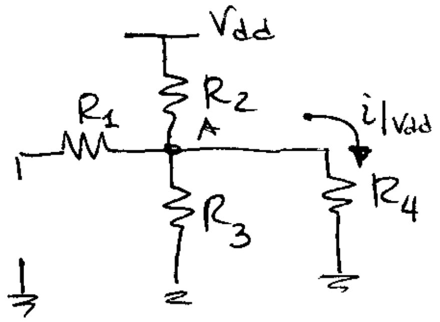
$R_1 = 400\,k\Omega$ $\quad$ $R_4 = 10\,k\Omega$

$R_2 = 500\,k\Omega$

$R_3 = 100\,k\Omega$

1. Let's calculate The current i flowing in The resistor R_4
2. Let's suppose To measure That current i with an amperometer with an internal resistance equal to $500\,\Omega$.
   Which is The True measured current? Which is The voltage drop across The amperometer?

1. The circuit is linear, Therefore we can apply The superposition Theorem.
   Let's switch off The current generator $I \Rightarrow$ it will be substituted by an open circuit



The current flowing in resistor R_2 is

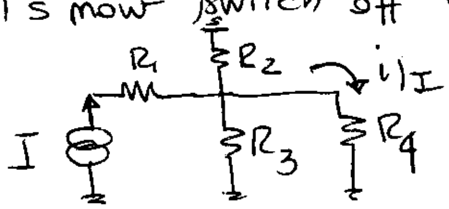
$$i_{R_2}\Big|_{V_{dd}} = \frac{V_{dd}}{R_2 + R_3 /\!/ R_4} = 9{,}82\,\mu A$$

By applying The current divider law at node A we find:

$$i\Big|_{V_{dd}} = i_{R_2}\Big|_{V_{dd}} \cdot \frac{R_3}{\underbrace{R_3 + R_4}_{\text{sum of The resistances of The Two branches}}} = 8{,}93\,\mu A$$

← resistance in which current doesn't flow

Let's now switch off The voltage generator V_{dd}



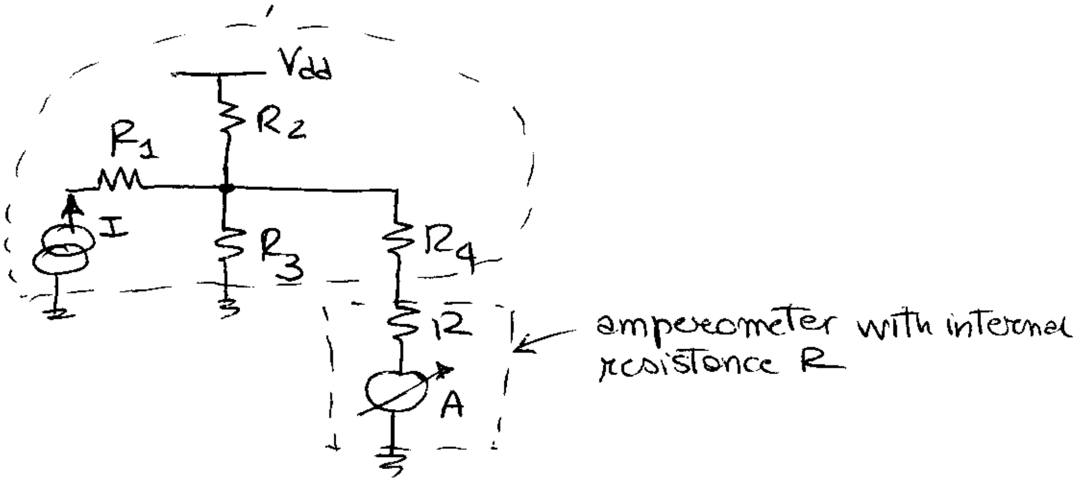
current divider law

$$i_I = I\,\frac{R_2 /\!/ R_3}{R_2 /\!/ R_3 + R_4} = 1.8\,mA$$

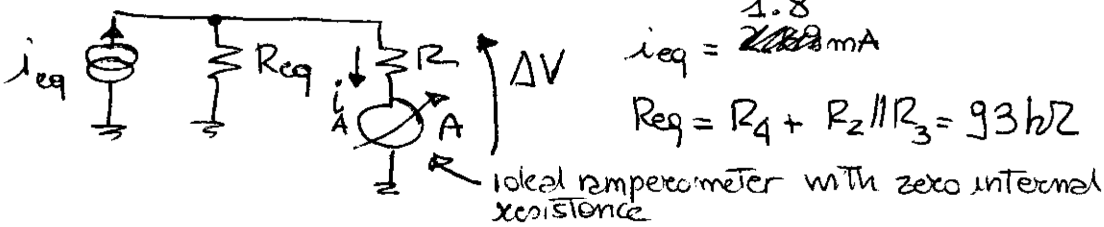
↳ by summing The Two contributions:

$i = i| + i| = 1.8\,mA + 893\,mA = 1.809\,mA$

2. An amperometer can be modeled as an ideal amperometer (i.e. with no internal resistance and therefore with voltage drop at its terminals equal to zero) with a series resistor of $R = 500\,\Omega$ in our case



Let's compute the Norton equivalent of the highlighted network
The "short circuit current is the one found before



$i_{eq} = 1.8\,\text{mA}$

$R_{eq} = R_4 + R_2 \parallel R_3 = 93\,k\Omega$

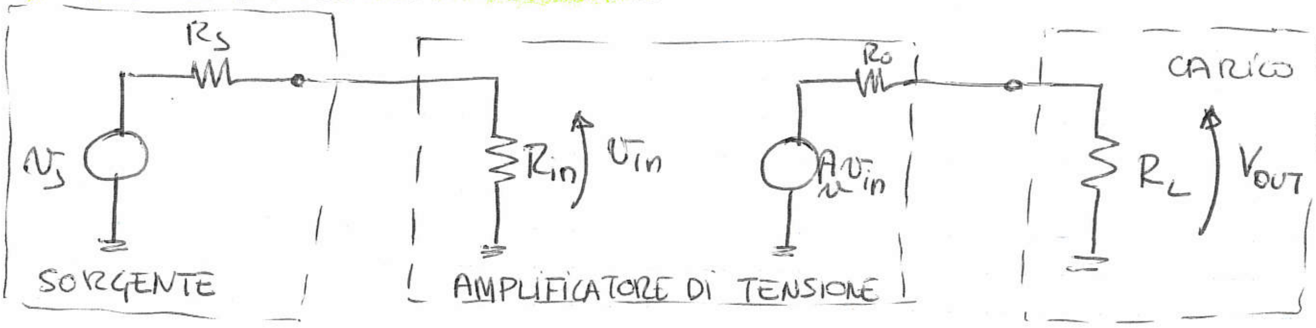
$$\Downarrow$$

$$i_A = \frac{R_{eq}}{R + R_{eq}}\, i_{eq} = \frac{93\,k\Omega}{93.5\,k\Omega} * 1.8\,\text{mA} = 1.79\,\text{mA}$$

$\Delta V = i_A * R = 0.89\,V$!! instead of the ideal 0V drop.

AMPLIFICATORI

* AMPLIFICATORE DI TENSIONE



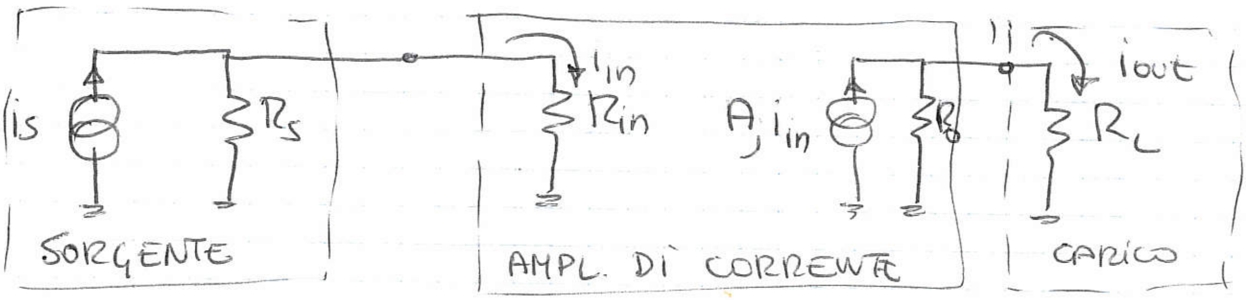
$$\frac{V_{OUT}}{v_S} = \underbrace{\frac{R_L}{R_L + R_O}}_{\text{partitore di uscita}} \cdot A_v \cdot \underbrace{\frac{R_{in}}{R_{in} + R_S}}_{\text{partitore di ingresso}}$$

$\Downarrow$

* al fine di leggere al meglio la tensione della sorgente:
  $R_{in} \gg R_S \Rightarrow$ BUON LETTORE DI TENSIONE

* al fine di erogare al meglio la tensione al carico
  $R_O \ll R_L \Rightarrow$ BUON GENERATORE DI TENSIONE

* AMPLIFICATORE DI CORRENTE



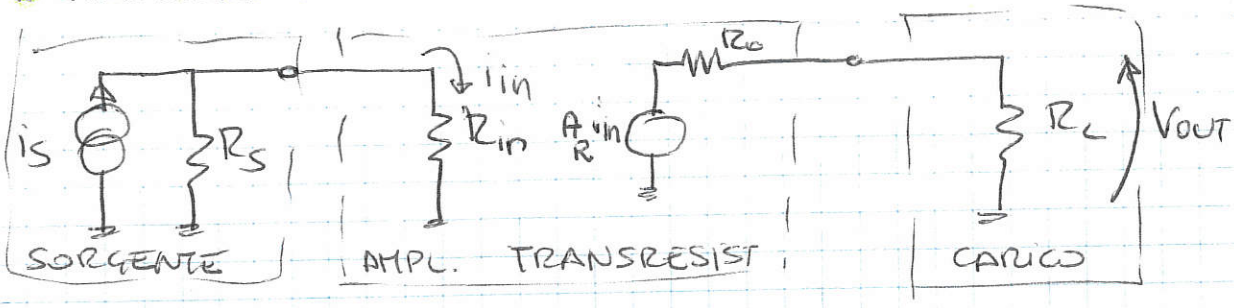
$$\frac{i_{OUT}}{i_S} = \underbrace{\frac{R_O}{R_O + R_L}}_{\text{partitore di uscita}} \cdot A_i \cdot \underbrace{\frac{R_S}{R_S + R_{in}}}_{\text{partitore di ingresso}}$$

$\Downarrow$

* BUON LETTORE DI CORRENTE $R_{in} \ll R_S$

* BUON GENERATORE DI CORRENTE $R_O \gg R_L$

* AMPLIFICATORE A TRANSRESISTENZA

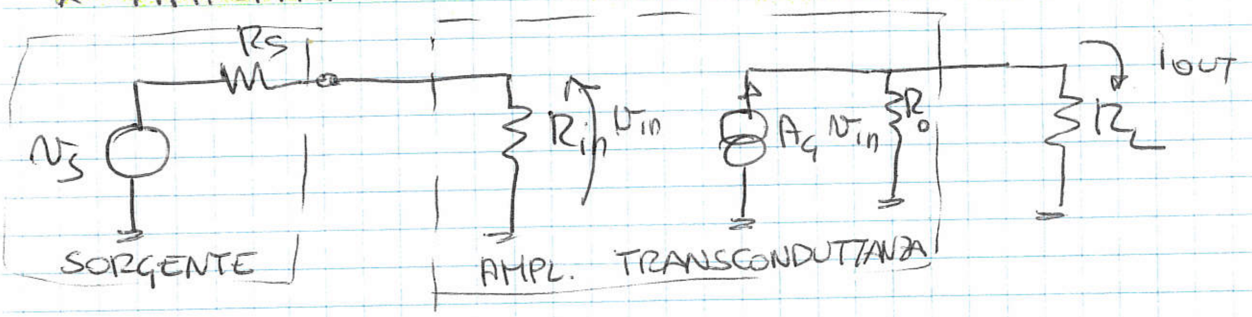


$$\frac{V_{OUT}}{i_S} = \underbrace{\frac{R_L}{R_L + R_o}}_{\text{partitore di uscita}} \cdot A_R \cdot \underbrace{\frac{R_S}{R_S + R_{in}}}_{\text{partitore di ingresso}}$$

* BUON LETTORE DI CORRENTE $R_{in} \ll R_S$

* BUON GENERATORE DI TENSIONE $R_o \ll R_L$

* AMPLIFICATORE A TRANSCONDUTTANZA



$$\frac{i_{OUT}}{v_S} = \underbrace{\frac{R_o}{R_o + R_L}}_{\text{partitore di uscita}} \cdot A_G \cdot \underbrace{\frac{R_{in}}{R_{in} + R_S}}_{\text{partitore di ingresso}}$$

* BUON LETTORE DI TENSIONE $R_{in} \gg R_S$

* BUON GENERATORE DI CORRENTE $R_o \gg R_L$

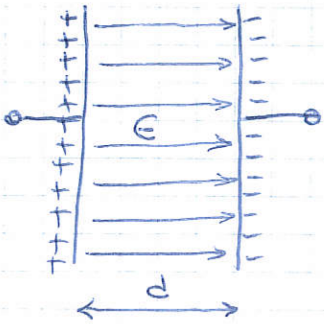
↳ ad esempio il Transistore MOSFET

RETI ELETTRICHE NEL DOMINIO DEL TEMPO

• CONDENSATORI

Una carica positiva $+Q$ deposta su una armatura, induce una carica $-Q$ sulla seconda armatura, nelle ipotesi di induzione completa.

Campo elettrico: applico il teorema di Gauss:

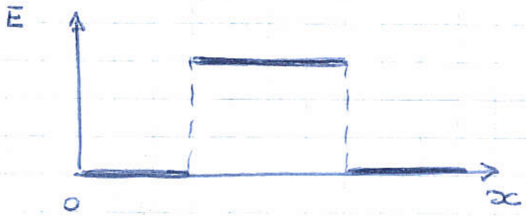


$$\int_{\Sigma} \vec{E} \cdot \vec{n} \, d\sigma = \int_{\Omega} \rho/\varepsilon \, dV$$

$$\Downarrow$$

$$E = \frac{Q}{\varepsilon A} = \frac{\sigma}{\varepsilon}$$

densità di carica sulla superficie $[C/cm^2]$



E

Tensione tra le armature: $\Delta V = E \cdot d = \frac{Q \cdot d}{\varepsilon A} = \frac{Q}{C}$

Capacità $\quad C = \frac{\varepsilon A}{d}$ [FARAD]

Se la carica Q varia nel tempo:

$$dQ = i(t)\,dt \Rightarrow dV(t) = \frac{dQ}{C} = i(t)\frac{dt}{C}$$

$$\Downarrow$$

$$i(t) = C \frac{dv(t)}{dt}$$

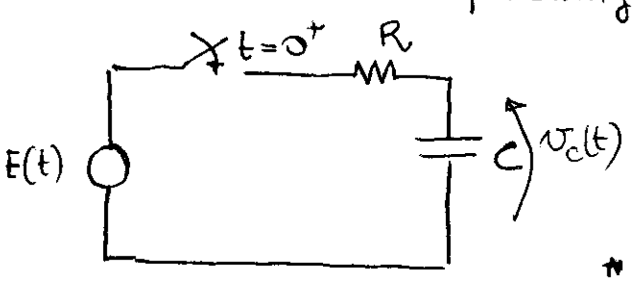
Valori tipici: $1pF \div 1000\mu F$ (tolleranza $\pm 20\%$)

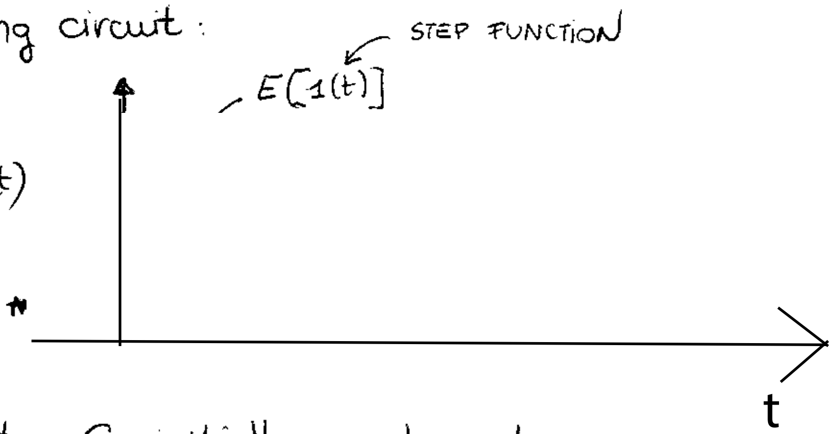
RC CIRCUIT IN THE TIME DOMAIN

- The Time evolution of a linear network with lumped elements is governed by a set of ~~linear~~ ordinary differential equations with constant coefficients and of order equal to the number of independent reactive elements present in the network.

- Capacitors (and inductors) are dependent when they can be reduced to a single element (series, parallel) or when their energetic condition is set by a condition (e.g. Three capacitors on the Three sides of a loop)

- Let's consider The following circuit:





- Let's consider The capacitor C initially uncharged
  ↳ $Q=0 \Rightarrow v_c(t)=0$

- at The beginning of The The Transient The current flowing in The resistor R is equal to

$$I=\frac{E}{R}=\frac{Q}{t}$$

and Therefore it charges the capacitor by accumulating The charge $Q=\frac{E}{R}t$ on its plates.

↳ $v_c(t) \simeq \frac{Q(t)}{C} \simeq \frac{E}{RC}t \quad$ for $t \ll RC$

- The voltage across The capacitor varies with time, Therefore also The current flowing in The resistor R varies with time
  ↳ The current charging The capacitor becomes smaller and smaller and The voltage across The capacitors builds up

more slowly, until The voltage across The resistor becomes zero and, Therefore, also The charging current

↳ v_c reaches The voltage E and saturates at That value.

⇩

Differential equation

$$v_c(t) = E - R\,i(t) = E - RC\,\frac{dv_c(t)}{dt}$$

⇩

$$v_c(t) = E\left[1 - \exp\left(-\frac{t}{RC}\right)\right] \underset{t \ll \tau}{\simeq} E\,\frac{t}{\tau} \qquad \tau = RC$$

RC CIRCUIT TIME CONSTANT

- RISE-TIME: time needed for The output voltage To go from 10% of its final value To 90% of it.

$$t_{rise_{10-90\%}} = 2.2\,\tau$$

- $t > 5\tau \Rightarrow \frac{E - v_c(t)}{E} < 1\%$

⇩ in order To know The Time evolution of The network for SINGLE TIME CONSTANT CIRCUITS (circuits composed of, or That can be reduced to, one reactive component and one resistor) it is enough to know The circuit time constant τ, The initial value of The output variable and The final value. The connection between The initial value and The final value is exponential with single Time constant equal To The circuit Time constant

METHOD OF ANALYSIS OF SINGLE TIME CONSTANT CIRCUITS

1. Evaluate The Time constant τ

   a. reduce excitation to zero
      - voltage source → short it
      - current source → open it

   b. find The equivalent resistor in parallel to The capacitor (or The equivalent capacitor in parallel to The resistor)

      * one single capacitor:
        - take out The capacitor
        - compute The equivalent resistors at The capacitor Terminals (R_{eq})

        ↳ $\tau = C R_{eq}$

      * more dependent capacitors:
        - take out The resistance
        - find The equivalent capacitor at The Terminals of The resistor (C_{eq})

        ↳ $\tau = R C_{eq}$

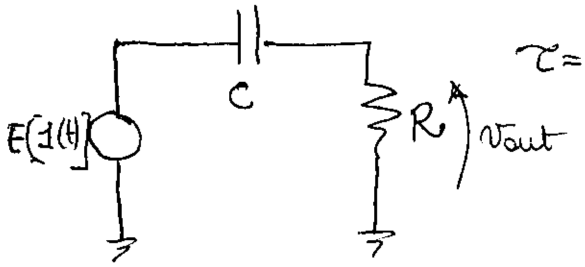
      * more Than one resistor and more Than one capacitor
        ↳ initial work to simplify The circuit

2. Compute The output variable at $t \to \infty$ → The input source is nearly constant → The capacitor does not let any current to flow Through it (i.e. open circuit)

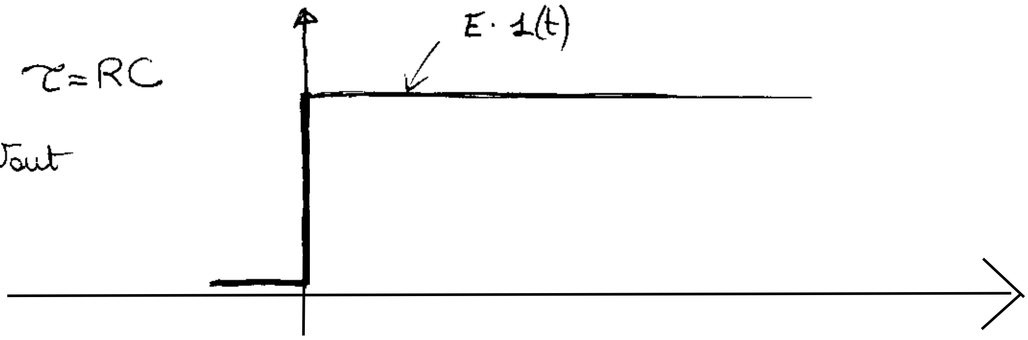
3. Compute The output variable at $t = 0^+$ → on The edge of The transition, The capacitor cannot change The voltage across its Terminals → it acts as a voltage source with The voltage difference of $t = 0^-$:

4. Connect The initial value and The final value with an exponential function with single Time constant equal to τ.

a/ CR CIRCUIT



$\tau = RC$



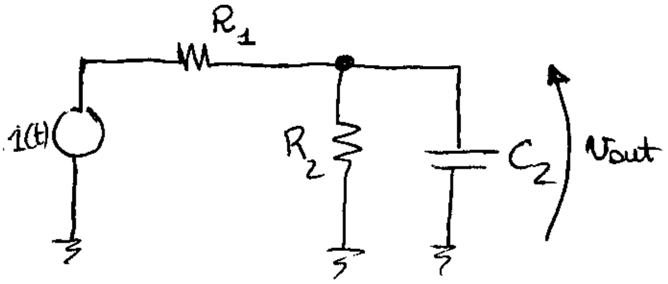
t

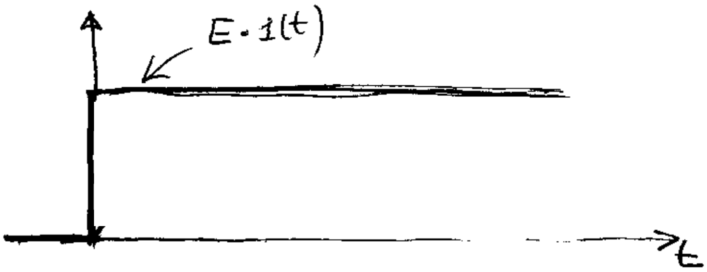
$t=0^+$: The capacitor cannot change the voltage drop at its terminal instantaneously
$\hookrightarrow \Delta V_c(0^-) = 0 = \Delta V_c(0^+)$
$\Downarrow$
$v_{out}(t=0^+) = E$

$t>0^+$: The capacitor is charged by the transient current flowing in the loop

$t \to \infty$ The capacitor acts as an open circuit $\Rightarrow i=0$; $v_{out}=0$

EXERCISE



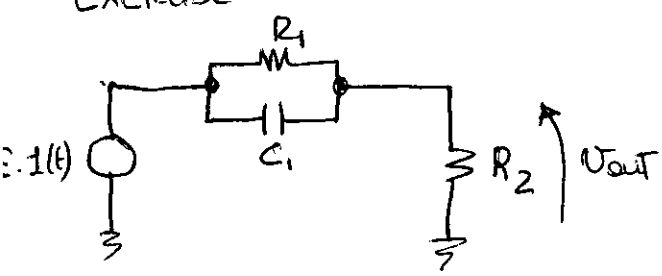


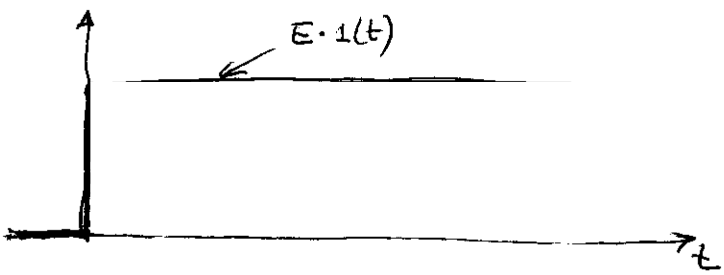
$t=0^+$: $\Delta V_c(0^-) = 0 = \Delta V_c(0^+) \Rightarrow v_{out}(0^+) = 0$

$t \to \infty$ C_2 open circuit $\rightarrow v_{out} = \frac{R_2}{R_1+R_2} E$

$\tau = C_2 * R_{eq} = C_2 (R_1 /\!/ R_2)$

EXERCISE



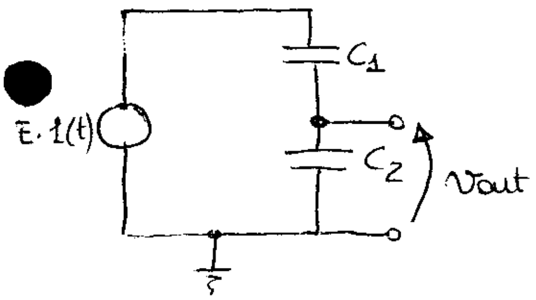


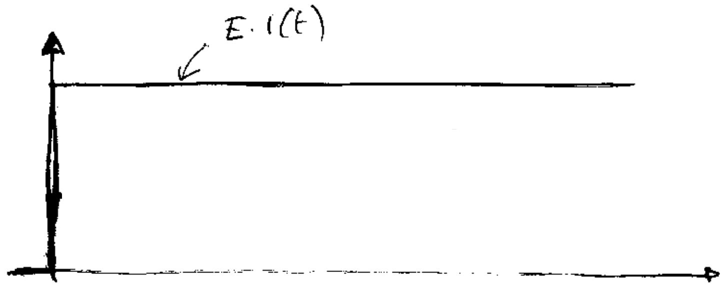
$\tau = C_1 * R_{eq} = C_1 (R_1 /\!/ R_2)$

$t=0^+$ $\Delta V_c(0^-) = 0 = \Delta V_c(0^+) \Rightarrow v_{out}(0^+) = E$

$t \to \infty$ C_1 open circuit $\rightarrow v_{out} = \frac{R_2}{R_1+R_2} E$

CAPACITIVE DIVIDER





C_1 and C_2 are in series

↳ $i_1 = i_2 \rightarrow C_1 \frac{dv_1}{dt} = C_2 \frac{dv_2}{dt}$

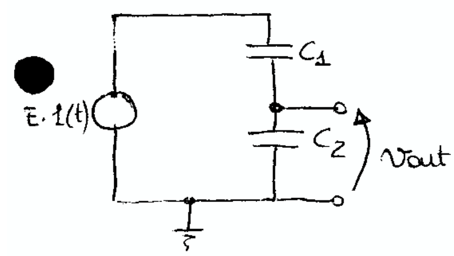
$$\Downarrow \quad \frac{C_1}{C_2} = \frac{dv_2}{dv_1} \rightarrow dv_2 = \frac{C_1}{C_2} dv_1$$

By integrating

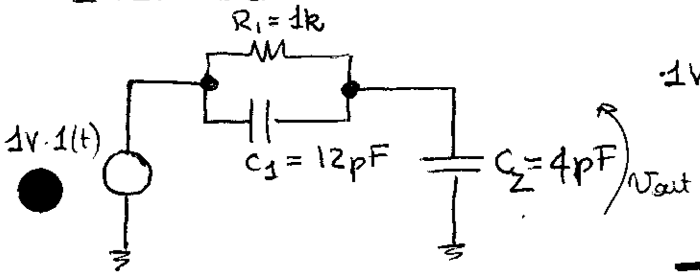
$$v_2 = \frac{C_1}{C_2} v_1 = \frac{C_1}{C_2}(E - v_2) = E\frac{C_1}{C_2} - \frac{C_1}{C_2} v_2$$

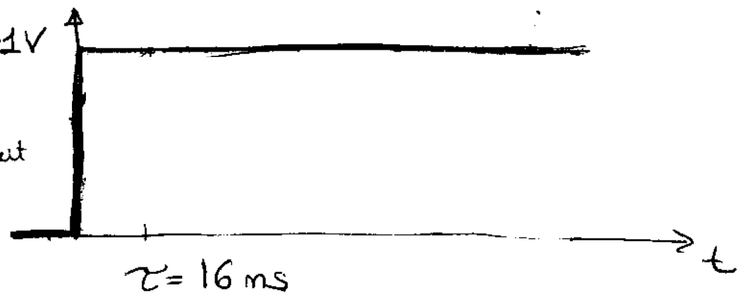
$$\Downarrow$$

$$v_{out} = v_2 = E \frac{C_1/C_2}{1 + \frac{C_1}{C_2}} = E \frac{C_1}{C_1 + C_2}$$



EXERCISE





$t = 0^+$: at the step the capacitors are dominating the behaviour of the network ⇒ capacitive divider

$$v_{out}(0^+) = \frac{C_1}{C_1 + C_2} 1V = \frac{12pF}{16pF} 1V = 0.75V$$

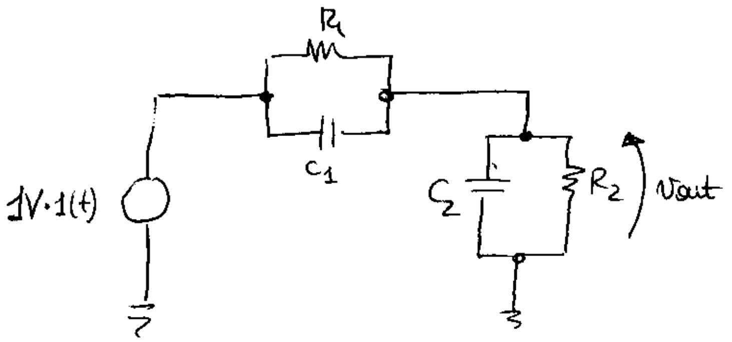
$t \rightarrow \infty$: C_1 and C_2 are open circuits → in R_1 no current can flow

↳ $v_{out} = 1V$

τ: The capacitors C_1 and C_2 are dependent (if we fix the charge deposited on the plates of one of the two and hence its voltage drop, the charge stored in the second is automatically set)

C_1 and C_2 are in parallel ⇒ $\tau = (C_1 + C_2)R_1 = 16$ ms

8/ COMPENSATED DIVIDER



$R_1 = 1k\Omega$

$R_2 = 1k\Omega$

$C_1 = 12pF$

$C_2 = 4pF$

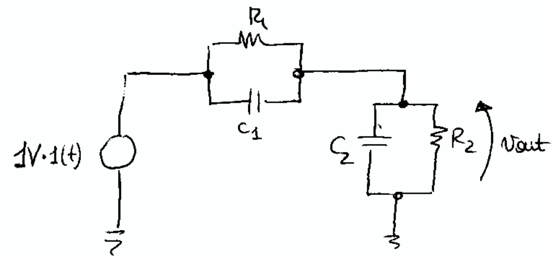
τ: The two capacitors are in parallel $\Rightarrow$ single time constant

$$\tau = (R_1 /\!/ R_2)(C_1 + C_2) = (1k /\!/ 1k)(12pF + 4pF) = 8ns$$

$t = 0^+$: The capacitors dominates the behaviour of the network

↳ capacitive divider.

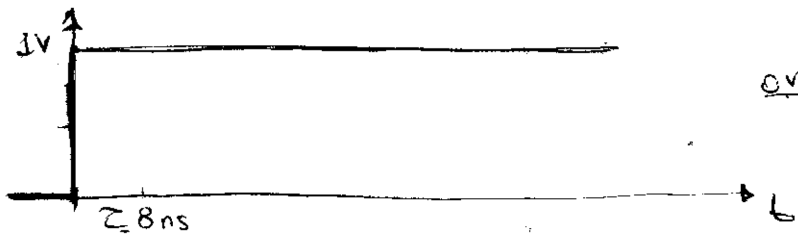
$$V_{out}(0^+) = \frac{C_1}{C_1 + C_2} 1V = \frac{12pF}{16pF} 1V = 0.75V$$



$t \to \infty$: The capacitors are open circuits

↳ resistive divider

$$V_{out}(t \to \infty) = \frac{R_2}{R_1 + R_2} 1V = 0.5V$$



OVER COMPENSATED DIVIDER

In order to have a perfect step as output voltage ;

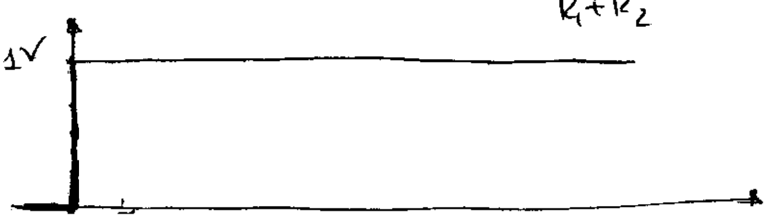
$$\frac{C_1}{C_1 + C_2} = \frac{R_2}{R_1 + R_2} \Rightarrow C_1R_1 + \cancel{C_1R_2} = R_2\cancel{C_1} + R_2C_2$$

COMPENSATED DIVIDER

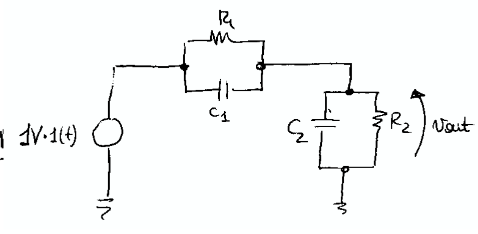
If $R_2C_2 < R_2C_2$ (e.g. $C_1 = 4pF$; $C_2 = 12pF$; $R_1 = R_2 = 1k\Omega$)

$t = 0^+ \Rightarrow V_{out}(0^+) = \frac{C_1}{C_1 + C_2} 1V = \frac{4}{16} 1V = 0.25V$

$$V_{out}(t \to \infty) = \frac{R_2}{R_1 + R_2} 1V = 0.5V$$

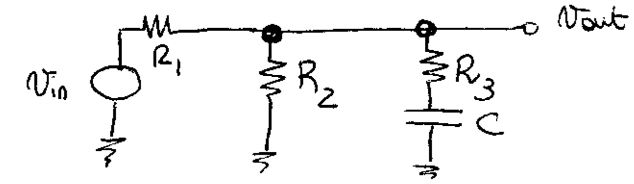


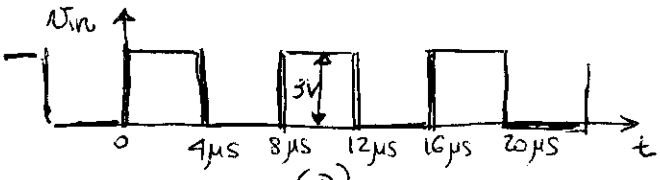
UNDE



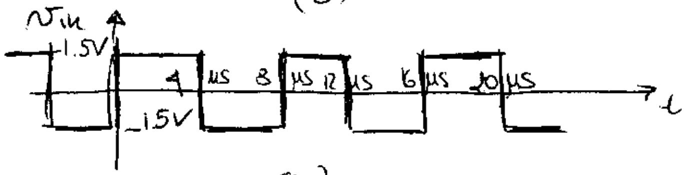
EXERCISE

Let's consider the following circuit:





(a)



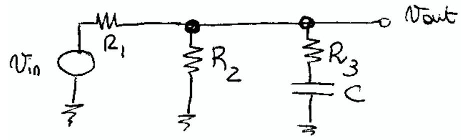
(b)

$R_1 = 2k\Omega$ $\quad R_2 = 6k\Omega$

$R_3 = 3k\Omega$ $\quad C = 50pF$

1. Draw the time diagram of the output voltage, v_{out}, quoting all the relevant points, when the input signal is the one shown in (a)

2. Draw the time diagram of the output voltage, v_{out} quoting all the relevant points, when the input signal is the one shown in (b)



It is a single time-constant circuit.
First of all, let's compute the circuit time constant:

$$\tau = C\left(R_3 + R_2 \| R_1\right) = 225\ ns$$

Therefore the output waveform has the time needed to reach the "final" voltage within every half period.

To determine the complete behaviour of the output voltage, let's compute the output voltage at the transition and its final value.

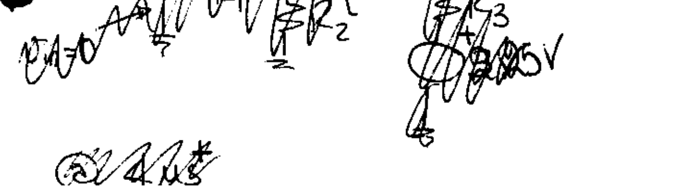
FINAL:

* when $v_{in} = 3V \Rightarrow$ final value $v_{out} = \frac{R_2}{R_1+R_2} v_{in,max} = \frac{6k\Omega}{8k\Omega} 3V = 2.25V$
* when $v_{in} = 0V \Rightarrow$ final value $v_{out} = \frac{R_2}{R_1+R_2} v_{in,min} = 0V$

Before the transition at $t=0^+$, $v_{out}(0^-) = 0V$ and $\Delta v_c(0^-) = 0V$ hence due to the constraint imposed by the capacitor $(\Delta v_c(0^+) = \Delta v_c(0^-))$

$$\rightarrow v_{out}(t=0^+) = \frac{R_3 \| R_2}{R_1 + R_3 \| R_2} v_{in,max} = \frac{2k\Omega}{4k\Omega} 3V = 1.5V$$

The same procedure applies to the transition at $t = 4\mu s$; before this transition $v_{out}(4\mu s^-) = 2.25V$ therefore $\Delta v_c(4\mu s^-) = 2.25V$
~~Due to the constraint imposed by the capacitor, at $t = 4\mu s^+$ the circuit becomes:~~

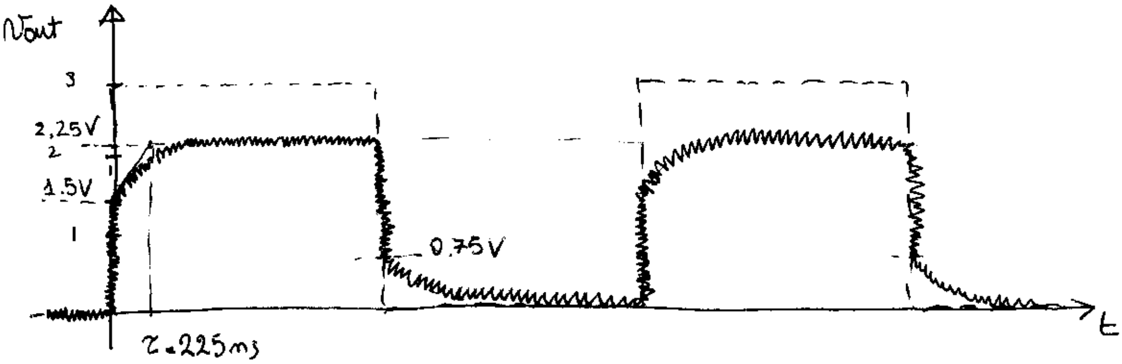


$= 0.75V$

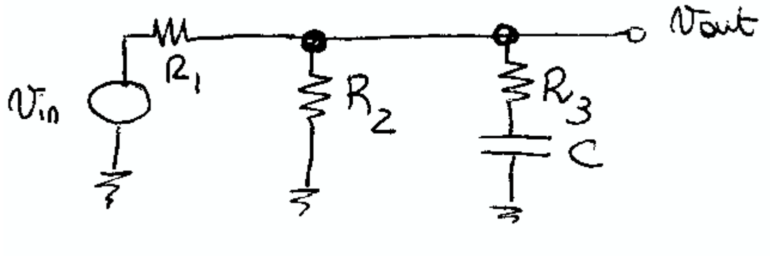
~~@ 4μs~~

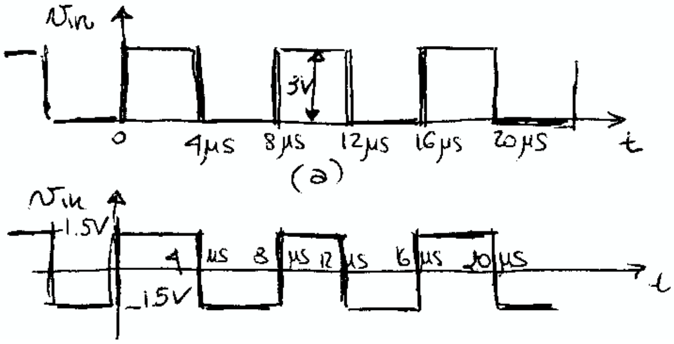
We can look at the input signal as showing a negative transition of 3V, therefore from the value of 2,25V we will have a step down of $\frac{R_3 \| R_2}{R_1 + R_3 \| R_2} * (-3V) = -1.5V$

The value of the output voltage at $t = 4\mu s^+$ will be $2,25V - 1,5V = 0.75V$



$= \quad \frac{\quad}{4} = \frac{\quad}{R_3 + R_1 \| R_2} = \frac{\quad}{4} \cdot \frac{\quad}{3 + \frac{12}{8}} = A \cdot 36$



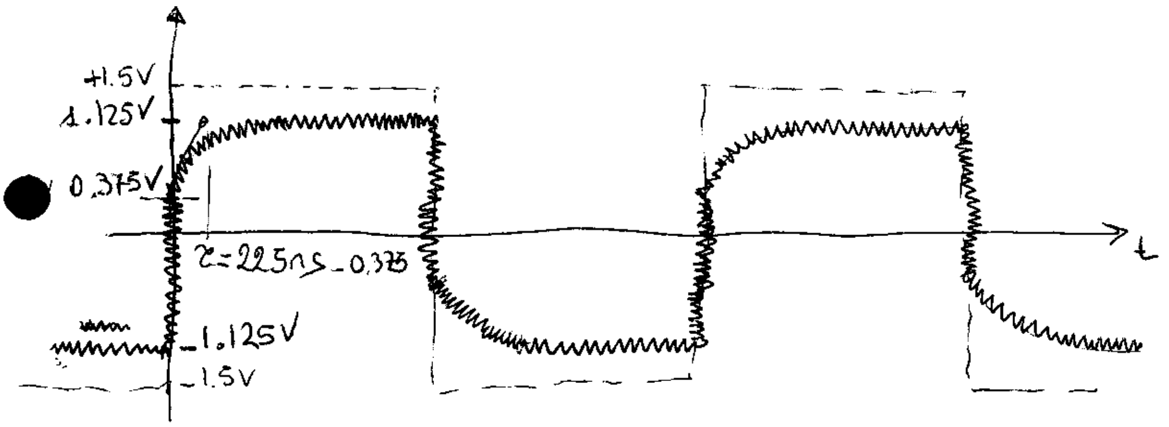


2. We can look to the square wave in (b) as the square wave shown in (a) plus a constant voltage of −1.5V. Since the circuit is linear we can apply the superposition theorem. We already know the output voltage when the square wave in (a) is applied. Therefore we have only to compute the contribution to the output of a DC voltage source of −1.5V. The capacitor will be, obviously, an open circuit.

$$v_{out}\Big|_{-1.5V} = -1.5V \cdot \frac{R_2}{R_1+R_2} = -1.125V$$

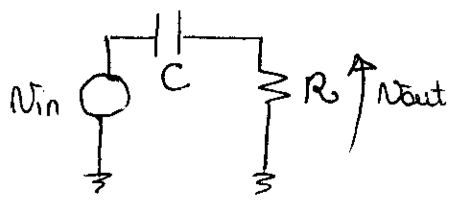
Therefore the output voltage, previously calculated, has to be shifted of −1.125V.

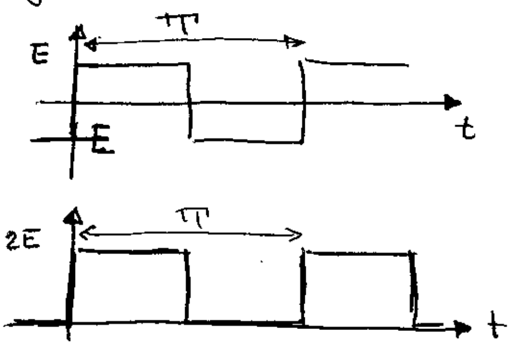
The resulting time diagram will be:



EXERCISE

Let's consider the following circuit:





(a) T = 10µs

E = 1V

$R = 2k\Omega$, $C = 100pF$

(b) T = 10µs

1. Draw the time diagram of the output voltage for v_{in} shown in (a)
2. Draw the time diagram of the output voltage v_{out} for v_{in} shown in (b)
3. Draw the time diagram of the output voltage v_{out}, for the input signal shown in (a) when the period of the square wave is 400ns. (Assume the circuit reached the steady state)

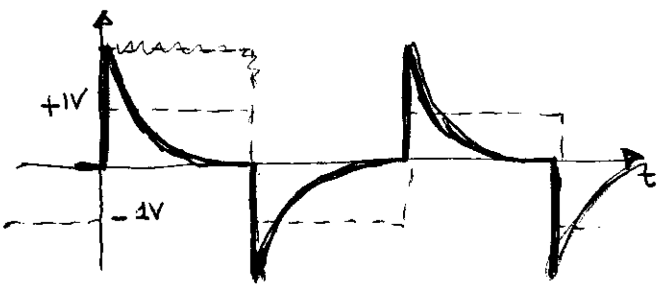
1. $\tau = RC = 200ns$ → The circuit reaches the steady state in each half period.

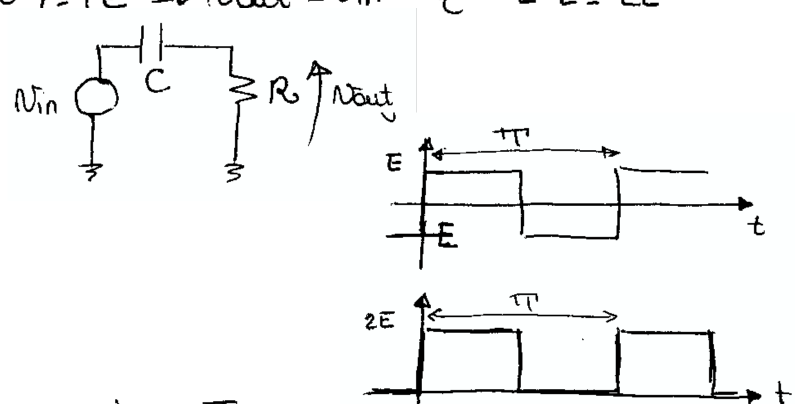
$t = 0^-$: $v_{out} = 0 \rightarrow \Delta V_C(0^-) = -E$

⇓ @ $t = 0^+$ $\Delta V_C(0^+) = \Delta V_C(0^-) = -E \Rightarrow v_{out} = v_{in} - \Delta V_C = E + E = 2E$

$t = 0^-$: $v_{out} = 0 \rightarrow \Delta V_C(0^-) = +E$

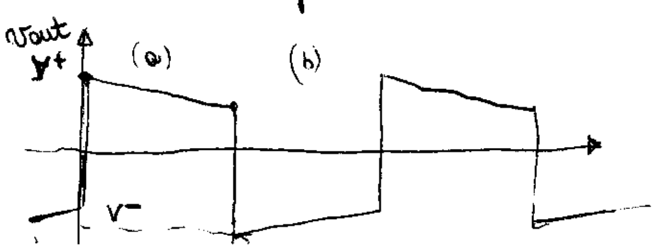
⇓ @ $t = 0^+$ $\Delta V_C(0^+) = \Delta V_C(0^-) = +E \Rightarrow v_{out} = v_{in} - \Delta V_C = -E - E = -2E$





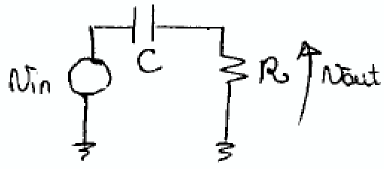
2. Again we can look at the input signal as the on... plus a DC constant voltage equal to the mean value of the signal in (b) (i.e. 1V)
The mean value is a DC voltage ⇒ it gives no contribution at the output of a CR network ⇒ the time diagram will be the same as in 1.

3. $\frac{T}{2} = \tau \Rightarrow$ The circuit do not saturate to the final value in each half period.



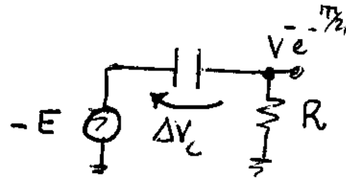
The output will resemble more a square wave.

$V^+ > 0$
$V^- < 0$



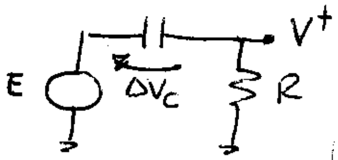
We have to compute the values V^+ and V^-. We can write the relationship of the voltage signal during half period (a) and (b) at the transiti

@ $t=0^+$



$\Delta V_C = -E - V^- e^{-T/2/\tau}$

@ $t=0^+$

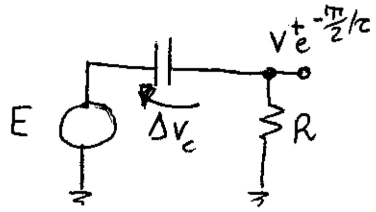


but $\Delta V_c(0^+) = \Delta V_c(0^-)$

$\Delta V_c(0^+) = E - V^+$

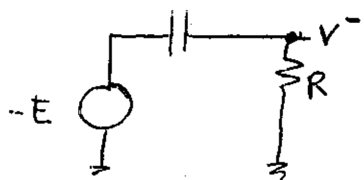
$\Rightarrow +E - V^+ = -E - V^- e^{-1}$

@ $t=0^-$



$\Delta V_c = E - V^+ e^{-T/2/\tau}$

@ $t=0^+$



$\Delta V_c(0^+) = -E - V^-$

but $\Delta V_c(0^+) = \Delta V_c(0^-)$

$\Rightarrow -E - V^- = E - V^+ e^{-1}$

$$\begin{cases} E - V^+ = -E - V^- e^{-1} \\ -E - V^- = E - V^+ e^{-1} \end{cases}$$

two equations in two unknown V^+ and V^-

$\Downarrow$

$V^+ = E + E + V^- e^{-1} = 2E + V^- e^{-1}$ (from 1st equation)

Inserting V^+ in 2nd equation:

$-E - V^- = E - (2E + V^- e^{-1}) e^{-1}$

$-V^-(1 - e^{-2}) = 2E - 2E e^{-1}$

$V^- = \dfrac{2E(e^{-1} - 1)}{1 - e^{-2}} = \dfrac{-2V \times 0.632}{0.865} = -1.46$

$\Rightarrow V^+ = 2V + (-1.46V)e^{-1} = 1.46$ (obviously: duty cycle 50% and zero average value)