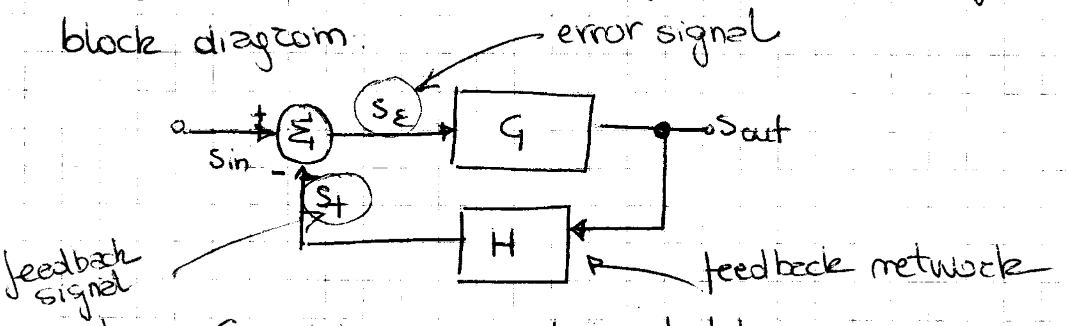
FEEDBACK THEORY

Feedback is a very important concept in electronics.
Feedback is a process in which information about the past or the present influences the same phenomenon in the present or future. As part of a chain of cause-and-effect that forms a circuit or loop, the event is said to "feed back" into itself.
The first use of feedback phrase was in the US in a patent on a mechanical system in 1860 and is an important topic in control theory and systems.
The use of the verb phrase feedback referred to electronic circuits is due to the Nobel Laureate Karl Ferdinand Braun but with a meaning different from the common meaning nowadays (undesired coupling between components of an electronic circuit) in 1909.
However the invention of negative feedback is due to Harold Stephen Black in 1927. In August he was commuting from New Jersey where he lived to New York City where he worked at The Bell Labs by taking a ferry to cross the Hudson River. Having nothing to write on, he sketched his thoughts on a misprinted page of The New York Times and then signed and dated it.
You can find the original paper on Negative Feedback published in 1934 reprinted in the Proceedings of IEEE, vol. 87, no 2, pp. 379-385, 1999.

In general it is referred to feedback with reference to the following block diagram:



Where G and H are unilateral blocks.

(2)

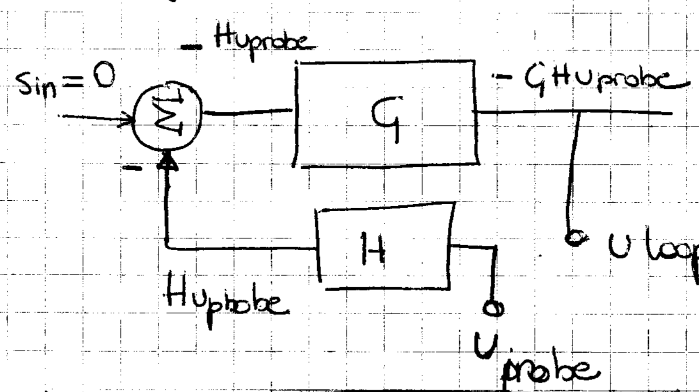
$$s_{out} = G s_{\varepsilon} = G(s_{in} - s_f) = G(s_{in} - H s_{out})$$

$$\hookrightarrow \boxed{\frac{s_{out}}{s_{in}} = \frac{G}{1 + GH}}$$ CLOSED-LOOP GAIN

LOOP GAIN

A very important quantity in a feedback system is the loop gain (G_{loop})

The loop gain is defined as the amplification of a test signal after "one lap around the feedback loop"



$$\hookrightarrow \boxed{G_{loop} = \frac{u_{loop}}{u_{probe}} = -GH}$$ LOOP GAIN

Under ideal conditions G should be infinite and hence G_{loop}.
The loop gain is a transfer function and can be positive or negative.

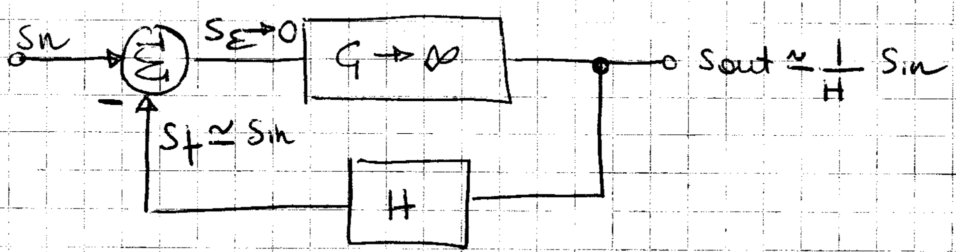
If the loop gain is negative, once we apply a positive step at the input, the loop returns a positive feedback signal s_f which is subtracted from the input stimulus.

$$\hookrightarrow s_{\varepsilon} = s_i - s_f < s_i$$

We will see that systems with positive feedback, on the contrary, tends to be unstable, but their inherent instability can be used to build particular circuits like comparators, oscillators, ...

We restrict our attention to negative feedback circuits.

GENERAL PROPERTIES OF NEGATIVE FEEDBACK CIRCUITS



1. $$\frac{S_{out}}{S_{in}} = \frac{G}{1+GH} \xrightarrow[G\to\infty]{} \frac{1}{H}$$ ASYMPTOTIC OR IDEAL GAIN does not depend on G, but only on the feedback factor H

↳ in order to have $G_{id} > 1 \Rightarrow H < 1$, i.e. the feedback network is, in general, an attenuator, that can be easily made with a high precision degree while the amplifying stage has to provide a very large gain

2. $$\frac{S_f}{S_{in}} = \frac{GH}{1+GH} \xrightarrow[G\to\infty]{} 1$$ The feedback signal is forced to be equal to the input signal

$$S_f = S_{out} H = \frac{GH}{1+GH} S_{in}$$

3. $$\frac{S_\varepsilon}{S_{in}} = \frac{1}{1+GH} \xrightarrow[G\to\infty]{} 0$$ The error signal tends to zero

4. $$\frac{dG_{cl}}{G_{cl}} = \frac{1}{1+GH} \frac{dG}{G}$$

However when referring to real circuits, it is difficult to highlight in the circuit the block responsible of amplification G and the feedback network. Moreover all the relationships have been obtained under the assumption of unilateral blocks, which is not the case.

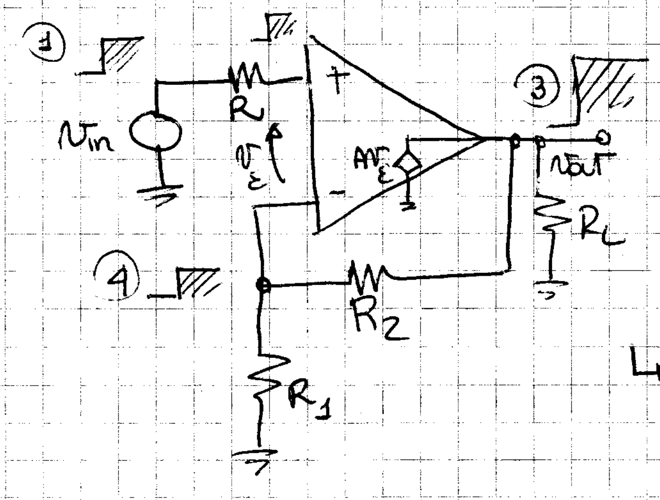
We can re-express the closed loop gain in terms of two quantities that can always be computed by circuit inspection

$$G_{closed\text{-}loop} = \frac{S_{out}}{S_{in}} = \frac{G}{1+GH} = \left(\frac{1}{H}\right) \frac{GH}{1+GH} = G_{id} \frac{-G_{loop}}{1-G_{loop}}$$

↳ $$\boxed{G_{closed\_loop} = G_{id} \frac{1}{1 - \frac{1}{G_{loop}}}}$$

COMPUTATION OF IDEAL GAIN IN REAL CIRCUITS

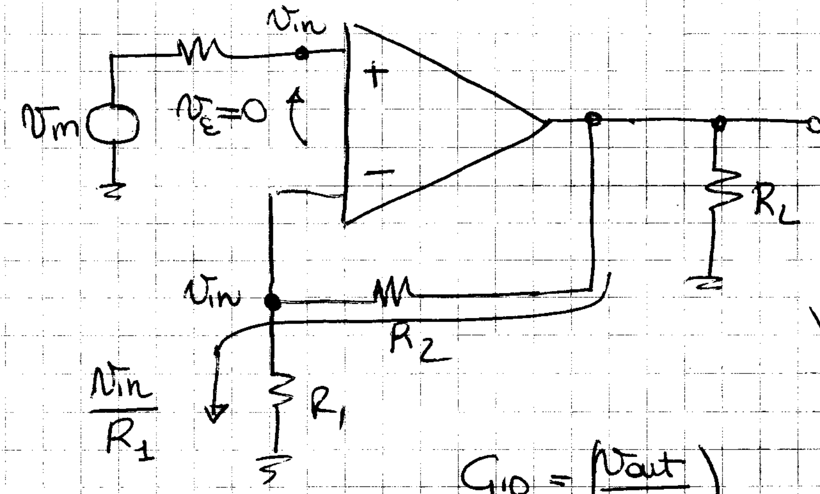
* NON INVERTING AMPLIFIER



1. A positive step is applied as input voltage.
2. The same step appears at the non inverting input since no current flow through R
3. The amplifier's output is forced to produce a positive step.
4. A positive step is FED-BACK to the inverting input of the opamp.

↳ $v_\varepsilon = v^+ - v^-$ is reduced by the action of the negative feedback.

If the loop-gain is high ⇒ v_ε tends to zero



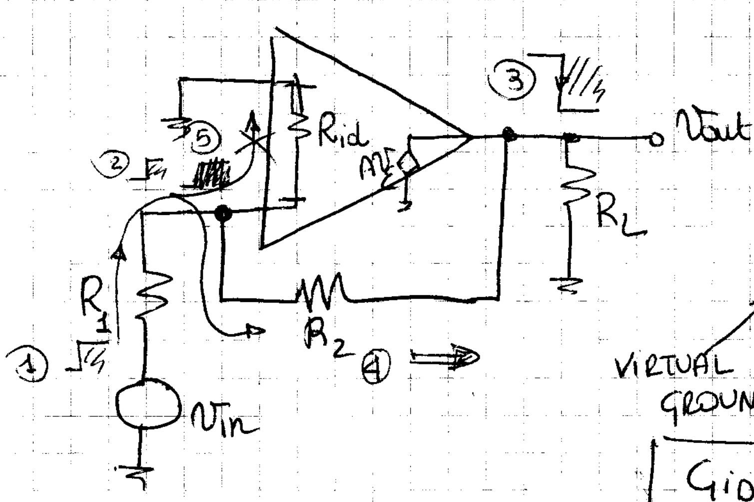
$$v_{out} = \frac{v_{in}}{R_1}(R_1+R_2) = v_{in}\left(1+\frac{R_2}{R_1}\right)$$

⇓

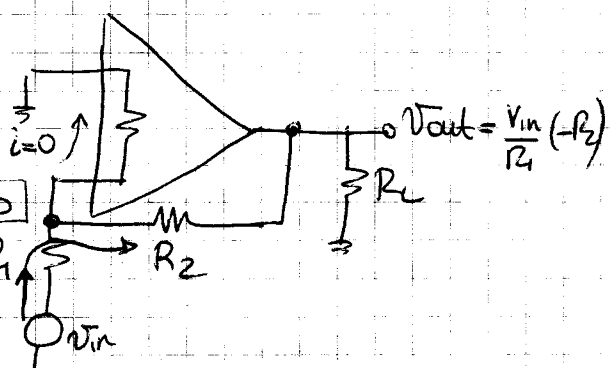
$$G_{ID} = 1 + \frac{R_2}{R_1}$$

$$G_{ID} = \left(\frac{v_{out}}{v_{in}}\right)_{v_\varepsilon = 0}$$

* INVERTING AMPLIFIER



⇒

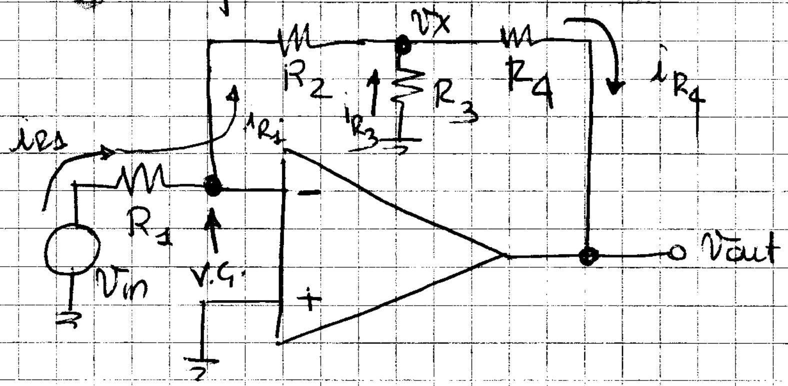


$\boxed{v^- = 0}$

VIRTUAL GROUND $\frac{v_{in}}{R_1}$

$$\boxed{G_{ID} = \frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}}$$

* T- feedback AMPLIFIER



$$i_{R_1} = \frac{v_{in}}{R_1}$$

$$v_x = - i_{R_1} * R_2 = - \frac{v_{in}}{R_1} * R_2$$

$$i_{R_3} = - \frac{v_x}{R_3} = \frac{v_{in}}{R_1} \frac{R_2}{R_3}$$

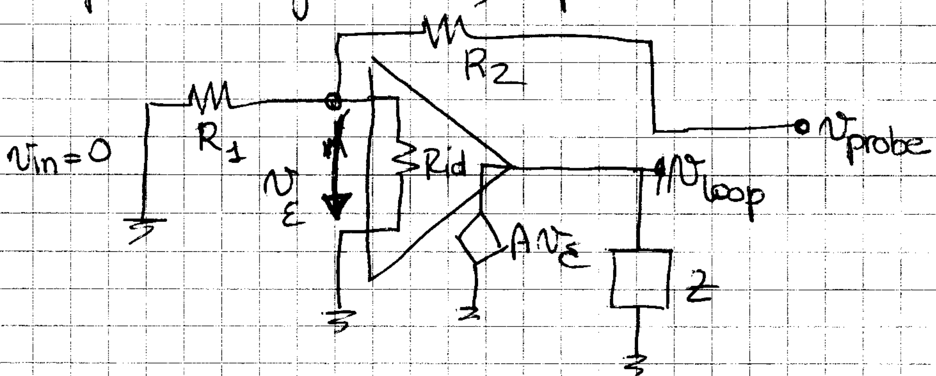
$$i_{R_4} = i_{R_1} + i_{R_3} = \frac{v_{in}}{R_1} + \frac{v_{in}}{R_1} \frac{R_2}{R_3} = \frac{v_{in}}{R_1}\left(1 + \frac{R_2}{R_3}\right)$$

$$v_{out} = v_x - i_{R_4} R_4 = - v_{in} \frac{R_2}{R_1} - \frac{R_4}{R_1} v_{in}\left(1 + \frac{R_2}{R_3}\right)$$

$$\hookrightarrow G_{ID} = \frac{v_{out}}{v_{in}} = - \frac{R_2}{R_1} - \frac{R_4}{R_1} - \frac{R_4}{R_1}\frac{R_2}{R_3}$$

COMPUTATION OF THE LOOP GAIN IN REAL CIRCUITS

We need to compute the gain for a signal doing one loop in the feedback loop.



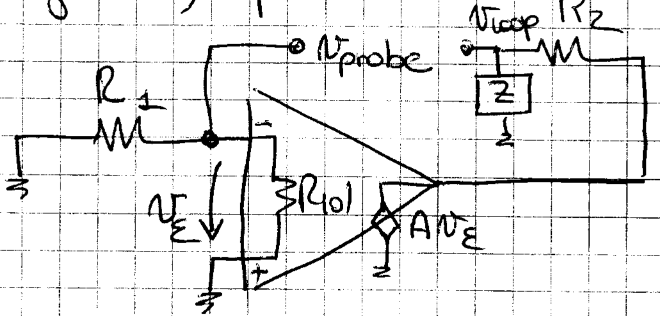
1. Switch off all the independent sources
2. Break the loop just after the dependent source and the rest of the circuit
3. Restore the original impedance
4. Drive the circuit with an independent voltage of the same

⑥ Type of The dependent one.

5. Find The output of The dependent source.

$$\Downarrow \quad G_{loop} = \frac{v_{loop}}{v_{probe}} = -\frac{R_1 \| R_{id}}{R_1 \| R_{id} + R_2} A$$

Notice That breaking The circuit loop alters The impedance level at The break point, Therefore it is requested To restore The original impedance



$Z = R_1 \| R_{id}$

$$v_{loop} = \frac{-Z}{Z+R_2} A v_{probe} = -\frac{R_1 \| R_{id}}{R_2 + R_1 \| R_{id}} A v_{probe}$$

Breaking The circuit right after The dependent source is a smart way to avoid The problem, since The output of The dependent source is The output of an ideal source That does not depend on The load.

EFFECT OF NEGATIVE FEEDBACK ~~ON~~ EQUIVALENT IMPEDANCES.

A general property of circuits with negative feedback is to STABILIZE* all the voltages of the nodes belonging to the loop and all the currents of the branches of the feedback loop.

* minimize the change in the voltages / currents of the feedback loop in response to an external source of perturbation

If the loop gain is infinite (i.e. ideal feedback), the change in the voltages/currents tends to zero.

The circuit reacts to the injection of a current in a node by changing everything permitted in the circuit so that the voltage of that node keeps constant.

The same holds for a voltage ~~source~~ in a branch of the loop.

The same concept can be seen in terms of equivalent impedances!

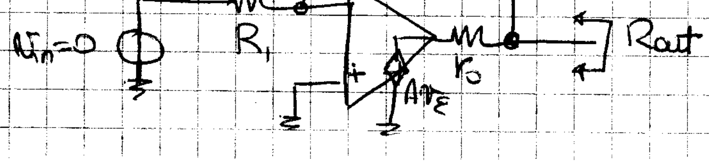
* The impedance seen by a test current source perturbing the voltage of a loop node, tends to zero:

$$R_{eq} = \frac{v_{eq}}{i_{eq}} \rightarrow 0 \quad \text{since } v_{eq} \rightarrow 0$$

* The impedance seen by a test voltage source perturbing the current ~~of~~ a loop branch, tends to infinity

$$R_{eq} = \frac{v_{eq}}{i_{eq}} = \frac{1}{\underbrace{\frac{i_{eq}}{v_{eq}}}_{\text{admittance seen by the voltage source}}} \rightarrow \frac{1}{0} = \infty$$

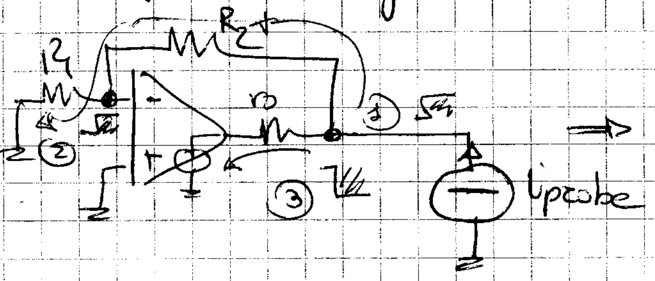
* OUTPUT IMPEDANCE AS AN EXAMPLE OF QUALITATIVE ANALYSIS



1. Set The input To zero.

2. choose The source To probe The impedance

We choose her a current soure because The feedback is controlling The voltage of That node and otherwise we kill The feedback



$\Rightarrow v_{out} \to 0$

$Z_{out} = \frac{v_{out}}{i_{probe}} \to 0$

COMPUTATION OF EQUIVALENT IMPEDANCES

∅. We need to know if we are perturbing a node ($\to$ voltage) or a branch ($\to$ current) of The loop To choose The appropriate source.

This can be easily done by computing The asymptotic value of The impedance seen from The test source when $G_{loop} \to \infty$.

- $Z \to 0$ $\Rightarrow$ excitation must be a Test current That tends to perTurb The voltage of a node.

$$Z = \frac{Z_0}{1 - G_{loop}}$$

open loop impedance seen when The output of The dependent source is zero.

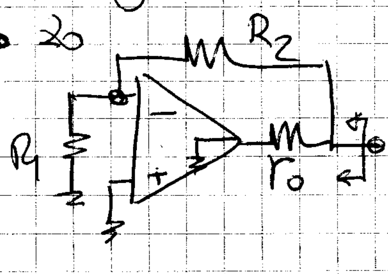
$$Z_0 = Z|_{A=0}$$

- $Z \to \infty$ $\Rightarrow$ excitation must be a Test voltage tending to perturb The current in a branch

$$Z = Z_0 (1 - G_{loop})$$

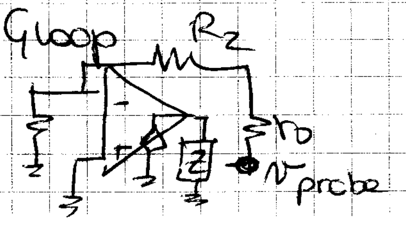
Returning to The output impedance calculation

- Z_0



$Z_0 = r_o \,//\, (R_1 + R_2)$

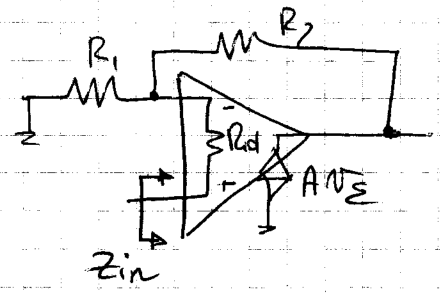
- G_{loop}



$$G_{loop} = \frac{-R_1}{R_1 + R_2 + r_o} A$$

$\hookrightarrow Z_{out} = r_o \,//\, (R_1 + R_2) \cdot \frac{1}{1 + AR_1/(R_1+R_2+r_o)}$

INPUT IMPEDANCE - NON INVERTING AMPLIFIER



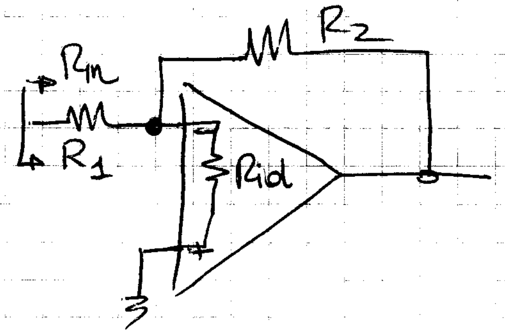
- ideal case $Z_{in} \to \infty$ (The feedback is stabilizing the current and therefore we need to use a voltage probe source)

- $Z_{in}^0 = R_{id} + R_1 // R_2$

- $G_{loop} = \dfrac{-R_1 // R_{id}}{R_2 + R_1 // R_{id}} A$

$\hookrightarrow Z_{in} = Z_0 (1 - G_{loop}) = (R_{id} + R_1 // R_2)\left(1 + A \dfrac{R_1 // R_{id}}{R_2 + R_1 // R_{id}}\right)$

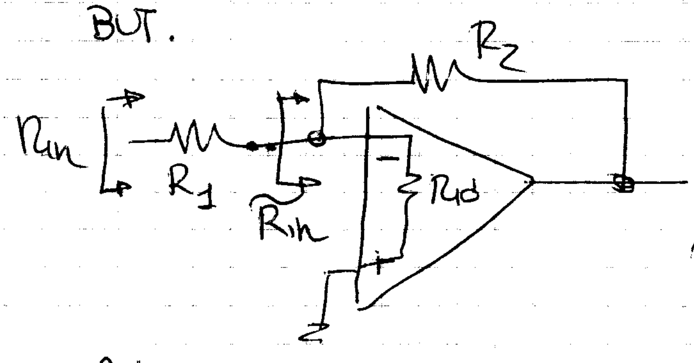
INPUT IMPEDANCE - INVERTING AMPLIFIER



- ideal case (i.e. $v_\varepsilon \to 0$) $\Rightarrow R_{in} = R_1$!

We are not perturbing a voltage or a current of the feedback loop, i.e. R_1 is outside the feedback loop!

BUT.



$R_{in} = R_{in} + \tilde{R}_{in}$

$\tilde{R}_{in}\big|_{ideal} \to 0$ (for $v_\varepsilon \to 0$)

- $\tilde{R}_{in}^0 = R_{id} // R_2$

- $\tilde{G}_{loop} = -\dfrac{R_{id}}{R_{id} + R_2} A$ (probe source is a current source at the inverting terminal)

$\hookrightarrow \tilde{R}_{in} = \tilde{R}_{in}^0 / (1 - \tilde{G}_{loop}) = \dfrac{R_{id} // R_2}{1 + \frac{R_{id}}{R_{id} + R_2} A}$

(10)

$$\Downarrow \; R_{in} = R_1 + \frac{R_2 \| R_{id}}{1 + A \dfrac{R_{id}}{R_{id} + R_2}}$$